Intergenerational Insurance

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Motivation

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NI 67, NI 64, LA RETRAITE :
C'EST 60 ANS !
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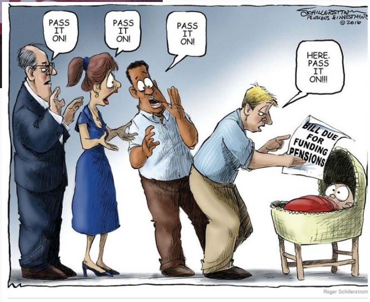
Motivation



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Pope says pension systems must not weigh on future generations

Reuters

Motivation

- Fiscal reforms may be difficult to implement
  - **Limited enforcement** due to political constraints
- When fiscal reforms involve different generations, a distributional **conflict** may arise
  - Conflict resolution may imply a higher fiscal burden on future generations
  - Welfare losses arise due to partial intergenerational insurance

Research Questions

I. How should **optimal intergenerational insurance** be structured under **limited enforcement** frictions?

II. What are the implications in terms of **risk-spreading** across generations?

What We Do

- We characterize **optimal intergenerational insurance** under **limited enforcement** when agents belong to finitely-lived **OLG**

... while the past literature has characterized:

i. Optimal intergenerational risk sharing under **full enforcement** (e.g., Aiyagari and Peled, 1991)

ii. Intergenerational risk sharing as a voting equilibrium, which is **not** necessarily optimal (e.g., Cooley and Soares, 1999)

iii. Optimal risk sharing under limited enforcement when agents are **infinitely lived** (e.g., Thomas and Worrall, 1988)

Mechanism in a Nutshell

A trade-off **Incentives** versus **Efficiency**:

✓ **Incentives**: Risk is partially spread onto future generations to provide incentives to the current generation to not walk away

⇒ Consumption depends on past shocks

✓ **Efficiency**: Period *resetting* in the provision of incentives to offset the welfare losses of shocks propagation

⇒ Consumption periodically resets to welfare maximising levels

Policy Implications

- Which policy institution may support the optimal allocation?
- A combination of **taxes**, **transfers** and **state-contingent bonds** can replicate the optimal allocation
- Limited enforcement implies a **non-linear fiscal reaction function** to public debt
  - $\Rightarrow$ A form of **fiscal fatigue** (Gosh et al 2013)
- Public **debt** as a **safe asset** (good hedge)
  - $\Rightarrow$ High sustainable debt even if expected primary surpluses are low (Brunnermeier et al 2022, Jiang et al 2022)

Presentation Outline

i. Model

ii. Full enforcement

iii. Limited enforcement

iv. Implications for debt sustainability

Model

- Discrete time $t = 0, 1, 2, \ldots, \infty$
- Two-period living agents: Young and Old
- Total endowment $e_t = e_t^y + e_t^o$ of perishable consumption good
- Aggregate (**growth**) risk $\gamma_t = \frac{e_t}{e_{t-1}}$
- Idiosyncratic (**distributional**) risk $s_t = \frac{e_t^y}{e_t} \in \{s(1), s(2)\}$ with $s(1) < s(2)$
- $\rho_t := (s_t, \gamma_t)$ with probability ϖ and $\rho^t := (\rho_1, \rho_2, ..., \rho_t) \in \mathcal{P}^t$

Model

- Young consumption $C(\rho^t)$ and old consumption $e_t - C(\rho^t)$
- Logarithmic per-period utility function $u(\cdot) = \log(\cdot)$
- Given homogenous utility we can consider the de-trended economy and characterize

$$\{c\} = \{c(\rho^t) = \frac{C(\rho^t)}{e_t} : t \geq 0, \rho^t \in \mathcal{P}^t\}$$

Planner Problem

- The planner chooses $\{c\}$ to maximize the **sum of expected discounted utility** of **all generations**

$$V\left(\{c\};\rho^t\right) = \underbrace{\frac{\beta}{\delta}\left(\log(1 - c(\rho^t))\right)}_{\text{Current Old}} + \underbrace{\mathbb{E}_t\left[\sum_{j=t}^{\infty}\delta^{t-j}U\left(\{c\};\rho^j\right)\right]}_{\text{Current Young and Fut. Gen.}}$$

where

$$U\left(\{c\};\rho^t\right) = \log(c(\rho^t)) + \beta\sum_{\rho_{t+1}}\varpi(\cdot,\rho_{t+1})\log(1-c(\cdot,\rho_{t+1}))$$

- **Participation constraints** of the **old**:

$$c(\rho^t) \leq s_t \tag{1}$$

- **Participation constraints** of the **young**:

$$U\left(\{c\};\rho^t\right) \geq \log(s_t) + \beta\sum_{\rho_{t+1}}\varpi(\cdot,\rho_{t+1})\left(\log(1-s_{t+1})\right) \tag{2}$$

Optimal Sustainable Intergenerational Insurance

- An Intergenerational Insurance rule is **sustainable** if the history-dependent consumption plan

$$\{c(\rho^t)\}_{t=0}^{\infty} \in \Lambda := \{\{c(\rho^t)\}_{t=0}^{\infty} \mid \text{(1) and (2)}\}$$

- A Sustainable Intergenerational Insurance rule is **optimal** if it maximizes $V\left(\{c\};\rho^t\right)$ subject to the constraint that the initial old receive a utility of at least ω_0:

$$\log(1 - c(\rho_0)) \geq \omega_0$$

Assumptions

Assumption (1)

The idiosyncratic and aggregate shocks are iid: $\varpi(\rho) = \pi(s)\varsigma(\gamma)$

$\Rightarrow$ Any sustainable intergenerational insurance rule $\{c\}$ depends **only** on the history of **idiosyncratic shocks**

Assumption (2)

$\beta \sum_r \pi(r) \frac{s}{1-r} > 1$

$\Rightarrow$ There **exists** a non-trivial sustainable intergenerational insurance that improves upon autarky

Recursive Formulation

- Let ω_r be the **state-contingent promise** to current young when next-period state is r and the promise to current old is ω
- The planner's optimization problem is:

$$V(s,\omega) = \max_{\{c,(\omega_r)_{r\in\mathcal{I}}\}} \frac{\beta}{\delta}\log(1-c) + \log(c) + \delta \sum_r \pi(r) V(r,\omega_r)$$

subject to

$$BC : \omega_{\min}(r) \leq \omega_r \leq \omega_{\max}(r)$$

$$PC_o : c \leq s$$

$$PC_y : \log(c) + \beta \sum_r \pi(r)\omega_r \geq \log(s) + \beta \sum_r \pi(r) \log(1-r)$$

$$PK : \log(1-c) \geq \omega$$

For This Presentation

Simplifying assumptions:

- $\beta = \delta$
- BC and PC_o are not binding

Notation:

- μ is the multiplier associated with PC_y
- λ is the multiplier associated with PK
- $c = \mathbf{c}(x)$ and $\omega_r = \mathbf{w}_r(x)$ with $x := (s, \omega)$ are the optimal consumption and state-contingent promised utility

Full Enforcement

- The Planner value $V(s, \omega)$ is subject to PK, but not PCy

- The first-order conditions are:

$$\frac{1 - \mathbf{c}(x)}{\mathbf{c}(x)} = 1 + \lambda(x) \quad \text{and} \quad V_\omega(r, \mathbf{w}_r(x)) = 0$$

  The envelope condition is:

$$V_\omega(x) = -\lambda(x)$$

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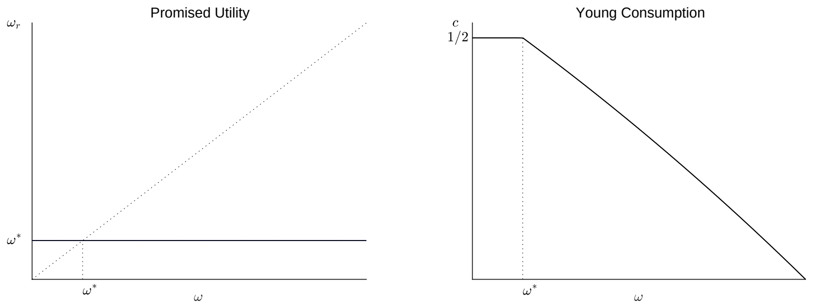
- There is $\omega^* = \log\left(\frac{1}{2}\right) = \sup\{\omega | V_\omega(x) = 0\}$

$\Rightarrow$ If $\omega_0 \leq \omega^*$ then $\lambda = 0$ and $c_0 = c^* := \mathbf{c}(s_0, \omega^*) = \frac{1}{2}$

$\Rightarrow$ If $\omega_0 > \omega^*$ then $\lambda > 0$ and $c_0 = \mathbf{c}(s_0, \omega_0) = 1 - \exp(\omega_0)$

$\Rightarrow$ $\mathbf{w}_r(s_0, \omega_0) = \omega^*$ for any ω_0, s_0, and r

Full Enforcement



Proposition

Under **full enforcement**, *the optimal allocation is* **stationary** *and the long-run distribution of* ω *is* **degenerate** *with mass at* ω^*

Limited Enforcement

- The Planner value $V(s, \omega)$ is subject to both PK and PCy

- The first-order conditions are:

$$\frac{1 - \mathbf{c}(x)}{\mathbf{c}(x)} = \frac{1 + \lambda(x)}{1 + \mu(x)} \quad \text{and} \quad V_\omega(r, \mathbf{w}_r(x)) = -\mu(x)$$

The envelope condition is:

$$V_\omega(x) = -\lambda(x)$$

$\Rightarrow$ Updating rule:

$$V_\omega(r, \mathbf{w}_r(x)) = -\lambda(r, \mathbf{w}_r(x)) = -\mu(x)$$

Limited Enforcement

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⇒ Updating rule:

$$V_\omega(r,\mathbf{w}_r(x)) = -\lambda(r,\mathbf{w}_r(x)) = -\mu(x)$$

- There is $\omega^0(s) = \sup\{\omega | V_\omega(s,\omega) = 0\} \leq \omega^* \ \forall s$

⇒ If $\omega_0 \leq \omega^0(s_0)$ then $\lambda = 0$ and $c_0 = \mathbf{c}(s_0, \omega^0(s_0)) \geq \frac{1}{2}$

⇒ If $\omega_0 > \omega^0(s_0)$ then $\lambda > 0$ and $c_0 = \mathbf{c}(s_0, \omega_0) = 1 - \exp(\omega_0)$

Dynamics of Promised Utility

- The Planner would like to promise $\omega^0(s)$ to the current young
- But if PC_y is binding, the current young will refuse it
- In this case, the Planner must promise more to relax PC_y of the current young
- A higher promised utility means that more consumption must be delivered to next-period old, which tightens PC_y of future young

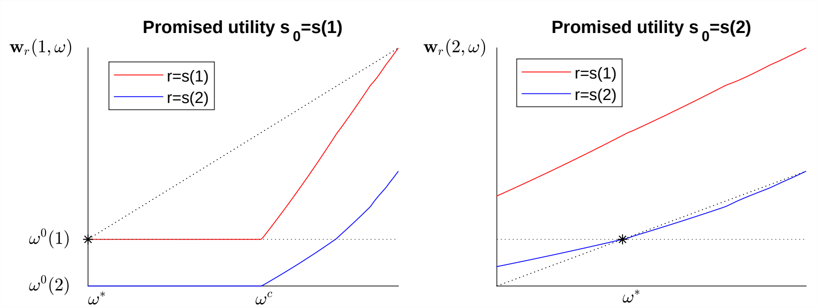
Dynamics of Promised Utility

Proposition

Assume that c^* violates PC_y in at least one s and $s(1) \leq c^*$, the optimal policy $\mathbf{w}_r(s, \omega)$ is

- *increasing in ω*
- *increasing in s*
- *decreasing in r*
- *there is a critical $\omega^c > \omega^0(1)$ such that $\mathbf{w}_r(1, \omega) = \omega^0(r)$ if $\omega \leq \omega^c$ (**resetting**)*
- *there is a unique fixed point $\omega^f(s) = \mathbf{w}_s(s, \omega^f(s)) = \omega^* \; \forall s$*

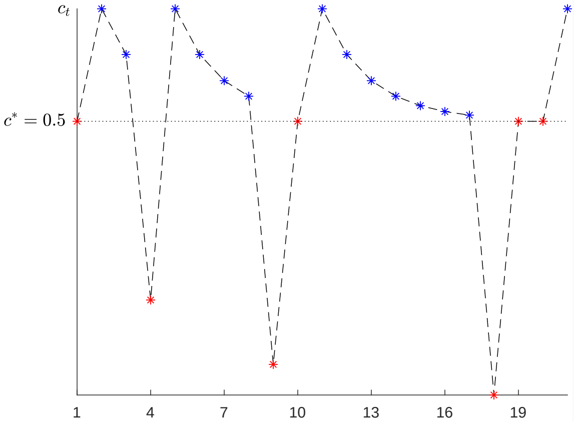
Dynamics of Promised Utility



Proposition

Under **limited enforcement***, the optimal allocation is* **history dependent** *and the long-run distribution of ω is non degenerate in an ergodic set with countable infinite states*

Dynamics of Consumption



Consumption of adjacent generations is serially **correlated** and does **not** follow a random walk

Stochastic Discount Factor

- In an equilibrium model, the SDF is

$$M_{t,t+1} = \beta \frac{u'(e_{t+1} - C(\rho^{t+1}))}{u'(C(\rho^t))}$$

- In the sustainable optimal allocation, the SDF is

$$M_{t,t+1} = \underbrace{\delta \cdot \frac{\mathbf{c}(x_t)/(1+\mu(x_t))}{\mathbf{c}(x_{t+1})/(1+\mu(x_{t+1}))}}_{m_{t,t+1}} \cdot \frac{e_t}{e_{t+1}}$$

Stochastic Discount Factor

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Proposition

- *Under* **full enforcement**, *the SDF is* $M^*_{t,t+1} = \delta \frac{e_t}{e_{t+1}}$.
- *Under* **limited enforcement**, *the SDF is decreasing in* γ_{t+1}, *increasing in* s_{t+1} *and decreasing (non-linearly) in* ω_t

Sustainable Public Debt

- The Planner uses one-period **state-contingent bond** $B_{r,t+1}$ in zero net supply and **taxes** $\mathcal{T}_t$ to repay **outstanding debt** D_t

$$D_t = \mathcal{T}_t + \sum_r q_{r,t+1} B_{r,t+1}$$

where **state prices** are

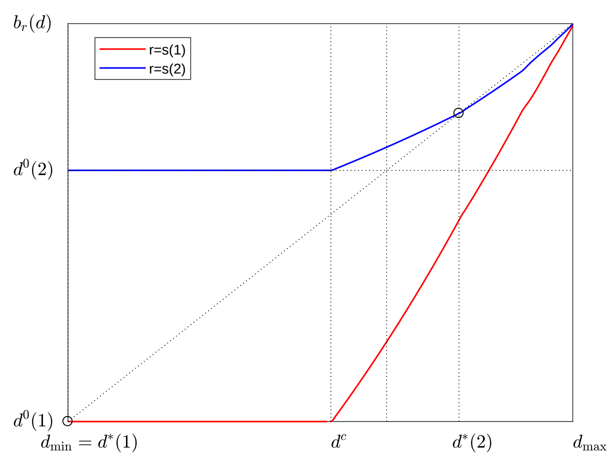
$$q_{r,t+1} := \pi(r) M_{t,t+1} = \pi(r) \beta \frac{u'(e^o_{t+1} + B_{r,t+1})}{u'(e^y_t - \mathcal{T}_t - \sum_r q_{r,t+1} B_{r,t+1})}$$

- Let $d_t := \frac{D_t}{e_t s_t}$, $\tau_t := \frac{\mathcal{T}_t}{e_t s_t}$ and $b_{r,t+1} := \frac{B_{r,t+1}}{e_{t+1} s_{t+1}}$

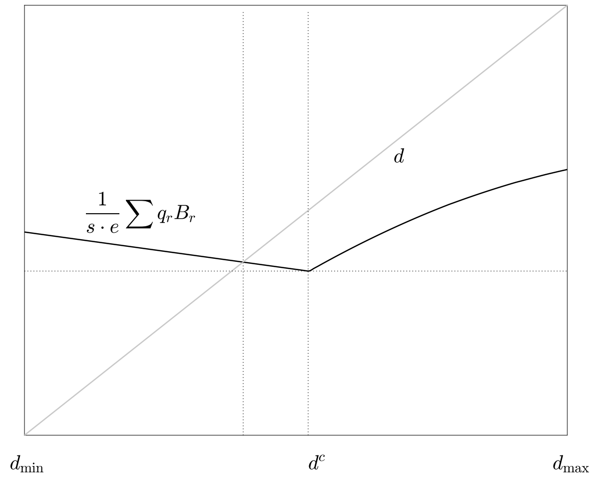
Dynamics of Debt

Dynamics of promises $\Longleftrightarrow$ Dynamics of debt

$$\omega_t = \log(1 - c_t) = \log(1 - s_t(1 - d_t)) \rightarrow d_t = \mathbf{d}(s_t, \omega_t)$$

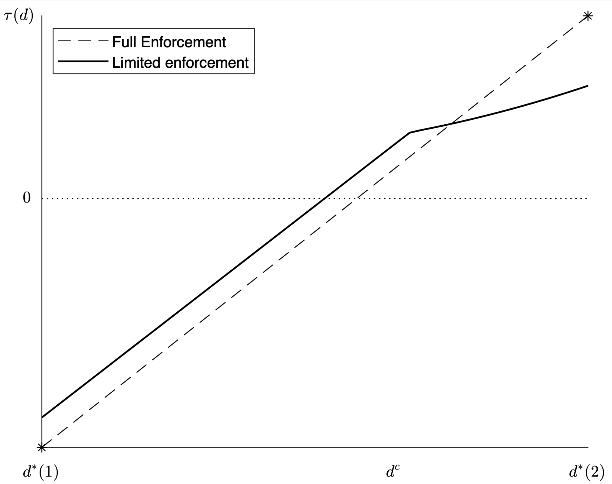


Bond Revenue



Below d^c bond revenue decreases since bond prices decrease and bond issuance is constant. Above d^c bond revenue can increase

Fiscal Reaction Function



The fiscal reaction function is non-linear in the outstanding debt: a form of fiscal-fatigue

Debt Valuation

- Under transversality condition, the budget constraint is

$$D_t = \overbrace{\mathcal{T}_t + \sum_{j=1}^{\infty} \mathbb{E}_t[M_{t,t+j}\mathcal{T}_{t+j}]}^{\text{NPV Primary Surpluses}}$$

where

$$\mathbb{E}_t[M_{t,t+j}\mathcal{T}_{t+j}] = \mathbb{E}_t[M_{t,t+j}] \cdot \mathbb{E}_t[\mathcal{T}_{t+j}] + COV_t[M_{t,t+j}, \mathcal{T}_{t+j}]$$

- If $COV_t[M_{t,t+j}, \mathcal{T}_{t+j}] > (\leq) 0$, then the sustainable debt is larger(lower) than the sum of future surpluses discounted at the risk free rate $\mathbb{E}_t[M_{t,t+j}]$

Debt Valuation

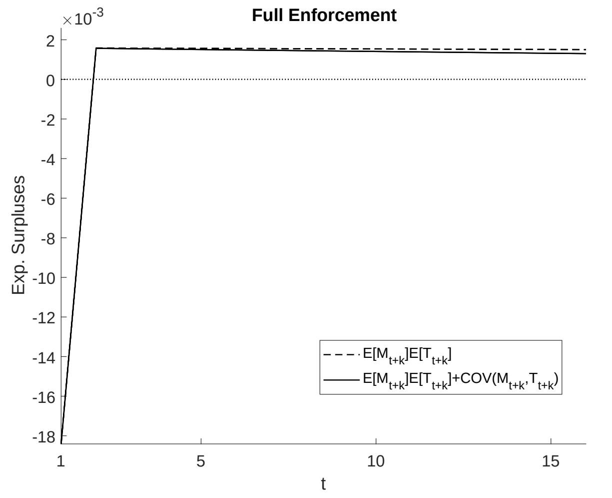
- Surplus $\mathcal{T}_{t+k}$ increases with both γ_{t+k} (pro-cyclical) and s_{t+k}
- The SDF $M_{t,t+k}$ decreases with γ_{t+k} (counter-cyclical) and increases with s_{t+k}

$\Rightarrow$ The $COV_t[M_{t,t+k}\mathcal{T}_{t+k}]$ can be decomposed in two terms

$$\underbrace{\mathbb{E}_t[m_{t,t+k}]\mathbb{E}[s_{t+k}\tau_{t+k}]\left(1-\mathbb{E}\left(\frac{1}{\gamma}\right)^k\mathbb{E}(\gamma)^k\right)}_{\geq<0}$$

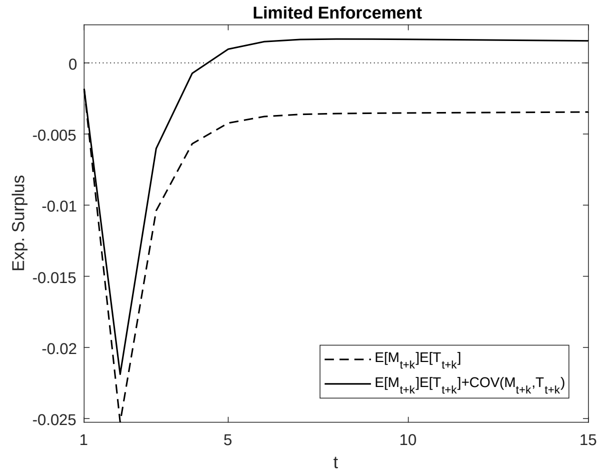
$$+\underbrace{\mathbb{E}_t[m_{t,t+k}s_{t+k}\tau_{t+k}]-\mathbb{E}_t[m_{t,t+k}]\mathbb{E}_t[s_{t+k}\tau_{t+k}]}_{>0}$$

Full Enforcement: $\dfrac{D_1}{e_1} = 0.10$



Under full enforcement, a positive amount of debt can be sustained only if expected future surpluses

Limited Enforcement: $\frac{D_1}{e_1} = 0.10$



Under limited enforcement, a positive amount of debt can be sustained even if expected future deficits (debt as good hedge)

Conclusions

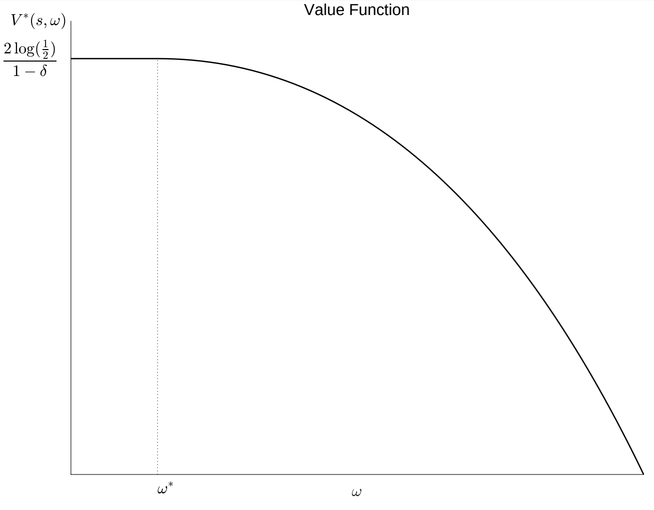
- We developed a theory of intergenerational insurance in a stochastic OLG model under limited enforcement
- The model implies that:
    i. Generational risk is spread across future generations because of the consecutive participation constraints
    ii. The optimum provides the basis for the design of a sustainable public debt

Conclusions

- We developed a theory of intergenerational insurance in a stochastic OLG model under limited enforcement
- The model implies that:
  i. Generational risk is spread across future generations because of the consecutive participation constraints
  ii. The optimum provides the basis for the design of a sustainable public debt
- Potential directions for future research:
  i. Richer demographic structure
  ii. Storage technology

Appendix

Value Function Under Full Enforcement



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