Partisan Conflicts, Public Debt and Growth*

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Abstract

We investigate the strategic nexus between productive spending, non-productive spending and public debt in the presence of partisan conflicts using a probabilistic voting framework where governments cannot commit to future policies. Partisan conflicts imply that, although parties share the objective of maximizing the probability of being elected, they disagree on either the optimal composition of government spending (productive versus non-productive spending) or the size of government. We find that the presence of partisan conflicts leads to a divergence of parties policy platforms and averts the economy to converge to an immiseration regime. In the Markov-perfect politico-economic equilibrium, the party that has a bias toward productive spending will turn out to be less disciplined than the party biased toward non productive spending, since higher growth will increase the endogenous debt limit and, in turn, will lead to more indebtedness and ultimately to less public consumption.


Keywords: Dynamic voting, Economic growth, Government debt, Intergenerational disagreement, Markov-perfect equilibrium, Partisan motive, Public good.

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1 Introduction

In democracies, policymakers are motivated by re-election prospects as well as by personal convictions or party ideology. Policy motivations drive decisions upon the size and composition of the welfare state, and in turn influence economic growth and the sustainability of public debt. Furthermore, an increase in political polarization between competing parties leads to more uncertainty. Then a natural question is: Does an increase of political disagreement result in inefficient policy reforms, such as excessive debt accumulation or under-investment in productive spending?

In the current paper, we investigate the strategic nexus between productive spending, non-productive spending and public debt in the presence of partisan conflicts, using a probabilistic voting framework in which governments cannot commit to future policies. Partisan conflicts imply that although parties share the objective of maximizing the probability of being elected, they disagree on either the optimal composition of government spending (productive versus non-productive spending) or the size of the government.

We model an overlapping generations world in which individuals live for three periods: young, adult, and elderly. They acquire skills during the first period, work and partially save from their income during the second period. When adult and old agents benefit from the provision of a non-productive public good (pork-barrel spending), while when young they benefit from the provision of public investments in education. When governments cannot issue public debt, we show in Lancia and Russo (2016) that productive investment, such as human capital, which benefits the young, expands the financing possibility of future non-productive public goods, which benefit the older cohorts. Hence, both productive and non-productive public spending can be sustained as part of the political economic equilibrium of a dynamic game played between successive governments, insofar as the voters can extract a rent through the exercise of political power when they become old. However, the result crucially relies on the assumption that the public budget is balanced in every period. In this paper, we explore how
productive and unproductive public spending are sustained by governments when they can issue public bonds.

We find that in the absence of partisan conflicts parties propose the same policy platform and the economy progressively depletes its resources through debt accumulation, leaving future generations enslaved, unless taxes are sufficiently distortionary. The reason for this is that both parties will finance productive spending which will stimulate growth as well as the provision of non-productive spending in the short run. However, higher growth will also expand the possibility of indebtedness, implying weaker fiscal discipline and higher public debt in the future. Higher public debt will then crowd out the future provision of any type of public spending. Each generation of voters will only endure a partial sacrifice in terms of reduced public consumption since the debt burden will be passed on to subsequent generations, until, in the long-run, income is fully expropriated to meet the debt obligations. In this scenario, the empowerment of young voters can discipline governments by reducing the accumulation of public debt, as in Song, Storesletten, and Zilibotti, 2012.

The presence of partisan conflicts leads to a divergence of the parties policy platforms and averts the economy's convergence to an immiseration regime. In this case, the political economy equilibrium is characterized by the following features: The party that has a bias in favor of financing productive spending will, once elected, devote a larger share of the public budget to productive spending relative to the party biased toward non-productive spending. This policy outcome will however only be sustainable in the short run. Indeed, a perpetual government led by the party biased toward productive spending will turn out to be less disciplined than a perpetual government led by the rival party, since higher growth will increase the endogenous debt limit, which in turn will lead to more indebtedness and ultimately to less public consumption. Hence, an implication of the model is that the empowerment of the young, who more strongly support the party that advocates productive spending, can actually lead to less fiscal discipline and more public debt in the long run.
In transition, equilibrium fiscal policies oscillate between periods of less public debt and more public spending and periods of less fiscal discipline. The length of these periods depends on an endogenous two-state Markov process, which describes the stationary probability of each party being in power. There exists a unique non-degenerate invariant distribution of fiscal policies, whose dispersion increases with the intensity of partisan conflicts.

We provide an analytical solution for the Markov perfect equilibrium of the voting game in a simple two-period economy and draw insights from the main results. A quantitative version of the model is then developed for predicting the impact of political frictions, as well as the empowerment of different age groups, on the dynamics and distribution of public debt and public spending.

The rest of the paper is organized as follows. The next Section reviews the related literature. Section 3 introduces the model. Section 4 presents the social planner solution. Section 5 studies the politico-economic equilibrium, while Section 6 provides an illustrative example. The final Section concludes and discusses challenges for future research.

2 Literature Review

This paper augments the literature on dynamic politico-economic models with overlapping generations that incorporates forward-looking decision makers in a multidimensional policy space (Krusell, Quadrini, and Rios-Rull, 1997). Related to government debt and economic growth, the literature can be divided into three strands.

The first strand refers to the literature on the political economy of debt initiated by Alesina and Tabellini (1990) and Persson and Svensson (1989), and further developed by Battaglini and Coate (2008), Caballero and Yared (2008), Aguiar and Amador (2011) and Song, Storesletten, and Zilibotti (2012). A common feature of these studies is the strategic use of public debt in economies where the interest rate is exogenous and governments with different preferences
alternate in power. We depart from this literature by recognizing the fundamental link between public debt and economic growth with endogenous turnover process.

The second strand contributes to the political economy literature of growth in which the government use the fiscal authority to alter the growth rate of the economy. Gonzales-Eiraz and Niepelt (2012) and Lancia and Russo (2015) have emphasized the strategic interdependence between productive and redistributive public expenditure as a social-policy-package deal. Unlike those papers, we focus on the dynamics of government debt, which is an essential element of macroeconomic fiscal policies and is ruled out in their model. Second, we introduce a dynamic electoral competition, which is not present in their work.

Finally, the present paper augments the literature on dynamic electoral competition. Azzimonti (2011) identifies in the incumbency bias the source of shortsightedness of government. In her framework, political competition reduces growth. Despite the complexity of the model, the author finds an analytical solution of the Markov-perfect equilibrium. This comes at the cost of some simplifying assumptions on the strategies of politicians. Under symmetric strategies, the two competing parties converge to the same platform. Thus, the exogenous incumbency bias is the only source of frictions in the parties’ reelection prospects. As main depart, we elucidate how politicians may endogenously manipulate the current political platform to increase the probability of winning the election. This mechanism generates results in line with the findings of Besley et al. (2010).

Closer to our spirit, Battaglini and Barseghyan (2013) explore the dynamic relationship between debt and growth. Policy choices are made by a legislature consisting of elected representatives. The current paper is different from their work in two important respects. First, it focuses on a small open economy to rule out disciplining mechanisms related to the strategic internalization of pecuniary externalities. Second, it introduce political frictions in the form of partisan politics, which exacerbate the intergenerational conflicts over the allocation of public

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1See Bai and Lagunoff (2011).
budget.

3 Economic and Fiscal Environment

Consider a small open economy inhabited by overlapping generations of three-period-lived – young, adult, and old – and ideologically heterogeneous agents who acquire skills in the first period, work in the second period, and live off savings in the last period. \( \kappa \in \{ a, o \} \) labels the adult and elderly cohorts, respectively. Time is discrete, indexed by \( t \), and runs from zero to infinity. At each time, two short-term mandate and partisan motivated parties, denoted by \( j \in \{ L, R \} \), run for office by proposing a political platform to maximize the probability of winning the election without commitment to future policies. Political interests are diverging in the preferred level and type of government expenditure.

**Production** At each time \( t \), the economy produces a single homogeneous private good, \( y_t \). Market production is subject to constant return and combines physical capital, \( k_t \), and effective labor demand, \( H_t \equiv L_t h_t \), according to the following technology:

\[
y_t = y \left( \frac{k_t}{h_t}, L_t \right) h_t = Ak_t^\kappa H_t^{1-\kappa}
\]

with \( L_t \) and \( h_t \) denoting the firms’ labor demand and human capital, respectively. Capital is perfectly mobile and depreciates fully after one period. We denote by \( R \) the world interest rate and by \( w \) the workers’ pre-tax wage per efficiency units. The human capital of an adult born at time \( t \) is produced according to a technology, which combines productive public transfers, \( I_t \), and parental human capital, \( h_t \), as complementary factors:

\[
h_{t+1} = h \left( I_t/h_t \right) h_t = ZI_t^\vartheta h_t^{1-\vartheta}
\]

(1)

where \( \vartheta \in [0, 1] \). A higher level of knowledge attained by one generation reduces the cost of the next generation to achieve the same level (intergenerational spillover). It follows that the
growth rate of human capital is equal to $h(I_t/h_t)$. 

**Household** An agent $i$ born at time $t-1$ and living at time $t$ evaluates private good $(c)$, public good $(G)$, and ideology according to the following additively separable utility function:

$$u(c^a_t) + \theta^a v(G_t) + \varsigma^a_i + \beta \mathbb{E}_{j^-} [u(c^a_{t+1}) + \theta^o v(G_{t+1}) + \varsigma^o_i]$$

(2)

where $\beta \in (0, 1)$ is the individual discount factor, and $\theta^\kappa$ denotes the preference weight on public goods for cohort $\kappa$. $\mathbb{E}_{j^-} [\cdot]$ is the expectation operator, which is conditional on the political platform implemented by the current incumbent party $j^- \in \{L, R\}$. The random variable $\varsigma^\kappa_i$ summarizes the utility derived by agent $i$ belonging to cohort $\kappa$ at time $t$ from political factors that are orthogonal to consumption (to be discussed later in more detail). The utility functions $u(\cdot)$ and $v(\cdot)$ are assumed to be homogenous, twice continuously differentiable, and with partial derivatives $u_c > 0$, $u_{cc} < 0$, $v_G > 0$, and $v_{GG} \leq 0$.

$c^a_t$ denotes the consumption at time $t$ when adult, and $c^o_{t+1}$ represents the consumption at time $t+1$ when old. In the first period of life, individuals do not consume. As adult, individuals elastically supply labor choosing the allocation of their time between market production, $l_t$, and household production, $1 - l_t$. They use their labor income, $wl_t h_t$, taxed at the rate, $\tau_t$, and home production, $F(l_t) h_t$, for consumption and saving, $s_t$. The household production is twice continuously differentiable function with $F_l < 0$, $F_{ll} \leq 0$, and $F(0) \geq 0$. Thus, the individual budget constraint for the adult agents is as follows:

$$c^a_t + s_t \leq w (1 - \tau_t) l_t h_t + F(l_t) h_t$$

(3)

Note that the government cannot tax household production. Hence, the taxation distorts the time agents work in the market. When old, agents retire and consume the sum of their savings, capitalized at a gross rate of return $R$, plus the revenue of the household production. Unlike adults, old agents are active only in the household production.\(^2\) Thus, the individual budget

\(^2\)The qualitative results are unchanged if one assumes that the agents receive no labor income in the second
constraint for the elderly agents is equal to:

\[ c_{t+1}^o \leq R s_t + F(0) \, h_t \]  

(4)

**Fiscal Constitution** At each date, short-term mandate governments, democratically elected by their constituents, use their fiscal authority to transfer income across different age groups. The allocation of public budget simultaneously serves the political scope of the elected representatives and the economic needs of their constituents. Revenue are raised by levying a proportional tax on labor income, \( \tau_t \), and by borrowing and lending in the bond market, \( \rho B_{t+1} \). We assume that there is a market of risk-free one-period bonds. Both citizens and the government have access to this market. \( \rho \) denotes the price of bonds – i.e., \( \rho \) dollar worth of bonds at time \( t \) yields one at time \( t + 1 \). The non-arbitrage condition requires that \( \rho = 1/R \). The fiscal revenues are used to provide unproductive public goods, \( G_t \), to finance productive investments, \( I_t \), and to acquire the previous period bond, \( B_t \). In every period, public budget implies that:

\[ TR(\tau_t, l_t h_t) + \rho B_{t+1} = G_t + I_t + B_t \]  

(5)

where \( TR(\tau_t, l_t h_t) = w \tau_t l_t h_t \) denotes the tax revenue function.\(^3\) The 4-tuple \( Q_t \equiv \{ \tau_t, I_t, G_t, B_{t+1} \} \) describes the collection of per-period fiscal bundle. Since governments have commitment of debt repayment, there exists an upper limit on the amount of issued bonds, \( \overline{B} \). Each government cannot borrow an amount \( B_{t+1} \), whose interest rate exceeds the maximum possible next-period tax revenues, i.e., \( \overline{B} = \max \frac{TR(t)}{1-\rho} \). Otherwise, it would be unable to repay the debt even without providing public goods or making investments. Moreover, the government is allowed to buy an amount of bonds that permit to finance the desired amount of unproductive public good and productive investments by means of the interest earnings. Then, there also exists a natural period. In the real world there are public transfers to the old from which we abstract for simplicity, and which affect the personal saving behavior.

\(^3\)We abstract from capital income taxation. However, in a small open economy this assumption is not particularly restrictive. Indeed, in the presence of capital income tax, assets would move after the tax announcement. Therefore, the tax rate in the political equilibrium would be necessarily zero.
lower bound of bond holding, that is \( B_t = -\max \frac{G_t + l_t}{1 - \rho} \). The initial level of government debt, \( B_0 \), belongs to such interval, \([B_t, B_t]\).

### 3.1 Competitive Economic Equilibrium

In a competitive economic equilibrium, each adult individual \( i \) chooses her labor supply and lifetime consumption, taking factor prices and fiscal policies as given. The equilibrium saving decision of an agent born at time \( t \geq 0 \) is given by:

\[
s_t(\tau_t) = \arg \max_{s_t > 0} \{ u(c_t) + \beta \mathbb{E}_{j > t} [u(c_{t+j})] \}
\]  

Eq. (6) implies that the present value of after-tax lifetime income is equal to \( C(l_t, \tau_t, h_t; R) \equiv w (1 - \tau_t) l_t h_t + F(l_t) h_t + \frac{F(0) h_t}{R} \). Therefore, the equilibrium labor supply is obtained as:

\[
l_t(\tau_t) = \arg \max_{l_t \in [0, 1]} C(l_t, \tau_t, h_t; R)
\]  

and it is equal to \( l_t(\tau_t) = -\frac{F^{-1}}{l_t} (w (1 - \tau_t)) \), where \( l_r \leq 0 \) and \( l_{\tau \tau} \leq 0 \). It follows that the top of the Laffer curve is equal to \( \tilde{\tau} = \arg \max_{\tau_t \in [0, 1]} w \tau_t l_t(\tau_t) h_t \). Note that, since labor supply only depends on the after-tax wage, there is no wealth effect. It is useful to define the marginal cost of public funds as \( MCPF(\tau_t) \equiv -\frac{C_t}{TR_{\tau_t}} \), which is increasing and convex in the tax-rate, and equal to 1 plus the deadweight loss of taxation. It measures the marginal cost of raising an additional unit of revenue via taxes. Therefore, it is a key measure of tax distortions in equilibrium in the presence of elastic labor supply: for any given tax rate, it is higher the more elastic is the labor supply.

Note that in this economy neither domestic savings nor domestic investments in physical capital are relevant state variables. This is due to two main reasons. First, because in a small open economy the world interest rate uniquely pins down the ratio of the factors of production. Second, because elderly voters do not receive government benefits in the form of private
consumption subsidies, which would otherwise partially substitute private savings.

**Definition 1 (Competitive Economic Equilibrium)** Given the initial conditions \( \{h_0, B_0\} \), a competitive economic equilibrium given public policies \( \{Q_t\}_{t=0}^{\infty} \) is a sequence of allocation \( \{c^a_t, c^e_t, s_t, l_t, k_t, h_{t+1}\}_{t=0}^{\infty} \), such that: (i) agents maximize utility subject to their budget constraints – i.e., Eqs. (7) and (6) are satisfied; (ii) firms maximize profits, so \( w = w(R) = (1 - \kappa) A \left( \frac{\kappa A}{R} \right)^{\frac{1}{1-\kappa}} \) and \( k_t = (\frac{\kappa A}{R})^{\frac{1}{1-\kappa}} L_t h_t \) with \( h_{t+1} = Z I_t^{0.5} h_t^{1-0}; \) and (iii) the labor market clears, i.e., \( l(\tau_t) = L_t \).

The indirect utility of any individual belonging to the adult and elderly cohort is, respectively, equal to:

\[
W^a(Q_t) \equiv \xi(\beta, R) u(C(\tau_t, h_t; R)) + \theta^a v(G_t) + \beta \mathbb{E}_{\tau_{t+1}} [\theta^a v(G_{t+1})] 
\]

where \( \xi(\beta, R) \) is a function of the exogenous interest rate and the individual discount factor, and

\[
W^o(Q_t) \equiv u(h_{t-1}; R) + \theta^o v(G_t)
\]

**Definition 2 (Equilibrium Feasible Allocation)** Given the initial conditions \( \{h_0, B_0\} \), an equilibrium feasible allocation is a sequence of competitive economic equilibrium allocations and public policies, \( \{c^a_t, c^e_t, s_t, l_t, k_t, h_{t+1}\}_{t=0}^{\infty} \) and \( \{Q_t\}_{t=0}^{\infty} \), that satisfy the balanced budget constraint, Eq. (5), and the fiscal feasibility conditions, at each time \( t \).

### 4 Social Planner Allocation

Before describing the equilibrium outcome under electoral competition, it is instructive to characterize the efficient allocation implemented by a Ramsey planner who maximize the discounted utility of all generations. As in Farhi and Werning (2007), the planner attaches a geometrically decaying Pareto weight, \( \delta \in (0, 1) \), to the discounted utility of each dynasty.
Varying $\delta$ yields all the allocation possibilities on the Pareto frontier. Given the initial level of assets held by agents ${h_0, B_0}$, the planner chooses the sequence ${\tau_t, G_t, I_t, B_{t+1}, h_{t+1}}_{t=0}^{\infty}$ that solves the following maximization program:

$$\max \sum_{t=0}^{\infty} \delta^t \left[ \xi(\beta, R) u(C(\tau_t, h_t; R)) + \theta^a v(G_t) + \beta \theta^o v(G_{t+1}) \right] + \frac{\beta}{\delta} \left[ u(c_0^t) + \theta^o v(G_0) \right]$$

subject to government budget constraint and human capital technology:

$$TR(\tau_t; h_t) + \rho B_{t+1} \geq G_t + B_t + I_t \quad (\delta^t \zeta_t)$$

$$h_{t+1} - h(I_t, h_t) \leq 0, \quad (\delta^{t+1} \varrho_{t+1})$$

where $(\delta^t \zeta_t)$ and $(\delta^{t+1} \varrho_{t+1})$ are the associated Lagrangian multipliers. Removing the functional arguments for expositional clarity, for each $t > 0$, the first-order conditions of the Lagrangian are equal to:

$$\tau_t : \xi(\beta, R) u_C(\tau_t) + \zeta_t TR_{\tau_t} = 0$$

$$G_t : \theta^a v_{G_t} + \frac{\beta}{\delta} \theta^o v_{G_t} - \zeta_t = 0$$

$$I_t : -\zeta_t + \varrho_{t+1} \delta h_{I_t} = 0$$

$$b_{t+1} : \rho \zeta_t - \delta \zeta_{t+1} = 0$$

$$h_{t+1} : \xi(\beta, R) u_{C_{h_{t+1}}} + \zeta_{t+1} TR_{h_{t+1}} - \varrho_{t+1} + \varrho_{t+2} \delta h_{h_{t+1}} = 0$$

together with the transversality conditions. Eliminating the multipliers, the following wedges for the optimal allocations must be satisfied:

$$\left( \theta^a + \frac{\beta}{\delta} \theta^o \right) v_{G_t} = \xi(\beta, R) u_C MCPF(\tau_t) \quad (10)$$
This first condition captures the *intra-temporal optimal redistribution wedge* between the private and public consumption. Higher distortionary taxation implies a smaller provision of public good. The redistributive allocation between the two sources of individual consumption is determined by the Pareto weight, the individual discounting, and the weights attached to the marginal utility of public goods by each cohort.

The following equation describes the *intertemporal productive wedge*, which internalizes the technological spillover of human capital investments, taking into account the distortionary effect of taxation.

\[
\frac{C_{h_{t+1}}}{MCPF(\tau_{t+1})} + TR_{h_{t+1}} = \frac{1}{\rho h_t} - \frac{h_{h_{t+1}}}{h_{h_{t+1}}} \quad (11)
\]

In the simple case of inelastic labor supply, Eq. (11) collapses to

\[
h_I = \frac{h_{I_{t+1}}}{\rho(C(0)h_{I_{t+1}}+h_{I_{t+1}})},
\]

where \( C(0) \equiv w + F(1) + \rho F(0) \) represents the net-present value of the individual endowment. Therefore, the intertemporal productive wedge quantifies the long-lasting impact of public investments. A benevolent social planner aims to maximize such positive technological spillover, whose impact outranges the current living generations.

The *inter-temporal optimal redistribution trade-off* is fully described by the following wedge:

\[
\rho v_G = \delta v_{G_{t+1}} \quad (12)
\]

If \( \rho = \delta \), then the social planner provides the same level of public good across different cohorts. Remarkably, when \( \delta < \rho \), in the long-run public debt approaches its upper bound, and, as a consequence, the public good provision converges to zero (immiseration regime). This results stems from the fact that the social planner with full commitment does not internalize the strategic valence of public debt, unlike policymakers (to be discussed later). In contrast, she determines the amount of bond by considering only the optimal consumption smoothing possibilities. Finally, notice that the optimal accumulation of human capital, as described in Eq. (11), does not affect the dynamics of public debt.
5 Politico-Economic Equilibrium

We characterize the politico-economic equilibrium of the economy as a subgame perfect equilibrium. Within each time period, the sequence of moves is as follows:

i. A new generation of young people is born.

ii. Before the realization of the ideological shocks among voters, candidates democratically compete by proposing their political platforms.

iii. All uncertainty is realized, and agents vote for their preferred candidates.

iv. The winning candidate implements the proposed political platform.

v. Agents save and supply labor and firms hire workers and rent capital.

vi. The older generation dies; the young and adult generations age and become adult and old, respectively.

Within a given period, the sequential politico-economic game can be viewed as Stackelberg, and it is solved by backward induction. This procedure entails the standard fixed-point problem, which nests two interdependent parts. First, given policies, adults determine the individual savings level and labor time, and firms produce the homogeneous final good, as described in Section 3.3 (competitive economic equilibrium). Second, to maximize the probability of winning the election, politicians promise voters an age-targeted fiscal bundle (politico-economic equilibrium). We assume that all expectations about subsequent events are correct and that all promises are honored. The fixed-point problem requires consistency of the laws of motion for policies that underlie the competitive economic equilibrium with the political selection.

5.1 Electoral Competition with Partisan Politics

In this Section, we present how citizens take their voting decisions and short-term mandate parties interact in electoral competitions under majoritarian rule.
Citizens The young have no political power. The utility derived from political factors, $\zeta_i^\kappa$, embeds two components: an idiosyncratic ideological bias, $\sigma_i^\kappa$, and an aggregate ideological bias, $\epsilon$. Formally, $\zeta_i^\kappa = (\sigma_i^\kappa + \epsilon) D^j$, where $D^j$ is an indicator function such that $D^R = 1$ and $D^L = 0$. A zero value of $\zeta_i^\kappa$ indicates the neutrality of voters’ ideology, whereas a positive value reveals that voter $i$ has a bias in favor of the candidate belonging to party $R$ compared to the candidate’s opponent.\(^4\) For simplicity, we assume that both shocks are i.i.d. over time.

The random component $\sigma_i^\kappa$ is drawn from a cohort-specific uniform distribution on the interval $[-1/(2\phi^\kappa), 1/(2\phi^\kappa)]$, where $\phi^\kappa$ denotes the distribution density. Note that, as the idiosyncratic bias is drawn from cohort-specific distributions, individuals belonging to the same cohort may vote differently. The additional random variable $\epsilon$ has a uniform distribution on the interval $[-1/(2\chi) + \psi^j - 1, 1/(2\chi) - \psi^j - 1]\) where $\chi$ denotes the density. Therefore, individual belonging to different cohorts may support the same party. The parameter $\psi^j$ captures the incumbent advantage, i.e., the ex-ante bias in favour of the previous period winning party. It is equal to $\psi$ if the incumbent $j$ is the party $R$ or $-\psi$ if the incumbent $j$ is the party $L$.\(^5\) At each time $t$, voter $i$ belonging to cohort $\kappa$ decides to support party $L$ if and only if $\tilde{\sigma}_i^\kappa - (\sigma_i^\kappa + \epsilon) \geq 0$, where $\tilde{\sigma}_i^\kappa \equiv W^\kappa (Q_i^L) - W^\kappa (Q_i^R)$, which represents the pivotal voter in cohort $\kappa$. Voting is not retrospective, as voters do not punish parties for their past performances, but they only evaluate them in terms of current and future expected policies. The total share of ballots cast in favor of party $L$ given the incumbent $j$ is equal to $n_t = 1/2 + \sum_\kappa \lambda^\kappa \phi^\kappa (\tilde{\sigma}_i^\kappa - \epsilon)$, with $\lambda^\kappa$ equal to the percentage of $\kappa$-cohort votes over the total number of votes, which can be interpreted as a proxy of the turnout rate at election of the adult and elderly cohorts. Hence, the probability that this party wins the election is:

$$p^{j - t}_i = \Pr (n_t \geq 1/2) = 1/2 - \psi^j - \frac{\chi}{\phi} \sum_\kappa \lambda^\kappa \phi^\kappa \tilde{\sigma}_i^\kappa$$ (13)

\(^4\)The random variable $\sigma_i^\kappa$ reflects the voters’ opinions about the candidates’ positions (e.g., civil rights, pro-market rules, religious issues) and personal characteristics (e.g., honesty, leadership, trustworthiness). Whereas, the additional random variable $\eta$ measures the average candidates’ popularity.

\(^5\)With this formulation, if the two parties decide to propose the same political platform, then the incumbent party will capture a fraction $1/2 + \psi$ of the votes, whereas the opponents will obtain the residual share $1/2 - \psi$. 

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where \( \phi \equiv \sum_{\kappa} \lambda^\kappa \phi^\kappa \) is the average density across cohorts. Note that, unlike static models of electoral competition with uncertainty, in this dynamic framework the current probability of winning the election is recursively affected by the next period probability, so as internalized by the current adult swing voter \( \tilde{\sigma}_t^a \), which will be called to cast their ballot also in the next-period election.

**Politicians** We now describe how short-term mandate forward-looking parties interact in the electoral competition. Public policies are chosen through a repeated voting system according to a simple majority rule. Therefore, the party that gains the largest number of votes win the election. Policy-makers are driven by two - not mutually exclusive - motives. First, they wish to win the election (office - seeking motive). Second, they wish to benefit their constituencies implementing a political platform consistent with their ideological position (partisan motive). Since parties have short-term mandate, they only aim to be elected in the current election to implement their preferred political platform. Nevertheless short-sighted politicians actually act in a long-sighted fashion. This is due to the fact that, as noted above, the current adult swing voter will also internalize the impact of current policies on the next-period policies and, indirectly, on the next-period probability of winning the election, as attached to the incoming incumbent party. Clearly, with homogeneous parties, the re-election probability – net of the incumbency bias – would be always one half, independently of the implemented platform. In contrast, by introducing partisan politics, we can explicitly model the endogenous changes in the re-election probabilities and the related strategic political feedback. This is fundamentally different from previous models of strategic use of policy instruments in which either parties are identical or a simple exogenous random election takes place in each period.\(^6\) Here, we model the partisan motive assuming that the two parties have diverging preferences over the composition of government expenditure. This seems particular natural in our environment, in which voters differ in their life-span. Reasonably, adult voters have stronger incentives in implementing productive

transfers compared to pork-barrel public goods. The opposite interests are defended by the old voters. Let $\alpha^j$ and $\gamma^j$ be a measure of the relevance of office-seeking and partisan motives, respectively, in the electoral competition. Then, the maximization program of party $\mathcal{L}$ is as follows:

$$\max_{Q_t} \gamma^c \varphi^L (I_t, G_t) + \alpha^c p^j_t$$

Analogously, party $\mathcal{R}$ solves the following problem:

$$\max_{Q_t} \gamma^r \varphi^R (I_t, G_t) + \alpha^r \left(1 - p^j_t\right)$$

The function $\varphi^j (\cdot)$ is a homogenous of degree one and twice continuously differentiable function. It satisfies the properties $\Delta_I \equiv \frac{\varphi^L}{\varphi^c} - \frac{\varphi^R}{\varphi^c} \geq 0$ and $\Delta_G \equiv \frac{\varphi^L}{\varphi^c} - \frac{\varphi^R}{\varphi^c} \geq 0$, which implies that party $\mathcal{L}$ cares at the margin weakly more of productive investments rather than of public good provision. The reverse is true for party $\mathcal{R}$. The degree of such disagreement is captured by the degree of polarization $\Delta_i$ with $i = I, G$: For $\Delta_i$ closer to zero, the more similar are the parties’ pliable policies on the $i$-policy dimension and the lower is the degree of party polarization. Similarly, if $\gamma^j = 0$ for each $j$, then the political mechanism would reduce to a static probabilistic voting model with both parties proposing the same platform, corresponding to the fiscal policy maximizing the welfare of current living voters. No candidate would be able to change current policies to obtain a net gain neither in terms of number of votes nor in terms of private benefits. In contrast, when $\gamma^j > 0$, the political platforms promised by the two parties are chosen with an eye toward the intergenerational special interests in different public spendings. Therefore, as it will become clear in the next Section, the two parties will necessarily propose different political platforms and richer strategic interactions will be generated.

Finally, note that our framework is quite flexible to explore how partisan politics affects the evolution of debt and growth both when parties disagree on the composition of public spending and when the parties’ disagreement involves the size of government. In the first case, $\gamma^L = \gamma^R$; whereas, in the second scenario, $\gamma^L \neq \gamma^R$. In the example economy, we will discuss the policy
implications of such different modelling assumptions.

5.2 Markov-Perfect Equilibrium

We now characterize the subgame perfect equilibrium of the voting game. At each time \( t \), the implementation of a political platform induces dynamic linkages of policies across periods through the evolution of the asset variables. Fully rational voters internalize such dynamic effects, which influence their strategic position over time. In principle, the construction of policies contingent on alternative histories and enforced by reputation mechanisms allows for multiple subgame perfect equilibria. We rule out such mechanisms and focus instead on differentiable stationary Markov policies as equilibrium refinement. Consecutive periods are linked by two state variables: the government debt, \( B_t \), the human capital stock, \( h_t \). Under homogenous preferences, we can reformulate the problem in per-efficiency units and identify the public debt per unit of human capital \( b_t = \frac{B_t}{h_t} \) as the only payoff-relevant state variable, with \( b_t \in [\bar{b}, \tilde{b}] \). Hence, the definition of policies in per-efficiency units is as \( q_t \equiv \{ \tau_t, \iota_t \equiv I_t, g_t \equiv G_t, b_{t+1} \equiv \frac{B_{t+1}}{h_{t+1}} \} \). A Markov-perfect equilibrium is formally characterized by the collection of (i) differentiable and stationary policy functions, \( Q^j (b_t) = \{ T^j (b_t), I^j (b_t), G^j (b_t), B^j (b_t) \} \), \( j \in \{ L, R \} \), (ii) the law of motion of the asset variable, \( h_{t+1}/h_t = H(I^j (b_t)) \),\(^\text{7}\) and (iii) the re-election probability \( P^j (b_t) \), defined as:

\[
P^j (b_t) = \frac{1}{2} - \psi^j + \sum_{\kappa} \lambda^\kappa \phi^\kappa \tilde{\sigma}^\kappa (b_t)
\]

where

\[
\tilde{\sigma}^\alpha (b_t) = \xi (\beta, R) \left( u \left( C \left( T^L (b_t) \right) \right) - u \left( C \left( T^R (b_t) \right) \right) + \theta^\alpha \left( v \left( G^C (b_t) \right) - v \left( G^R (b_t) \right) \right) \right) + \beta \delta^\alpha \left[ E_{L} \left[ v \left( G^j \left( B^L (b_t) \right) H \left( \iota^L_t \right) \right) \right] - E_{R} \left[ v \left( G^j \left( B^R (b_t) \right) H \left( \iota^R_t \right) \right) \right] \right]
\]

\(^\text{7}\)Note that, since the physical capital is not a relevant-payoff state variable, we are not interested in characterizing the functional equation that determines the future capital as a function of current states.
and
\[
\hat{\sigma}_o (b_t) = \theta_o \left( v \left( \mathcal{G}^L (b_t) \right) - v \left( \mathcal{G}^R (b_t) \right) \right)
\]
denote the adult and elderly swing voters, respectively. The adult pivotal voter is identified by the policies chosen by parties and also by the next-period re-election probability. As those variables are function of the state variable, so it is the re-election probability. As long as the two parties propose different electoral platforms because motivated by their partisan leanings, \( \mathcal{P}^j (b_t) \) is a non trivial function of the state variable. Moreover, notice that the pliable policy proposed by the winning candidate during the next-period election will depend on the political decision taken by the current incumbent party. It implies that, although parties have short-term mandate and are not directly interested to maintain power in the long-run, they strategically use their fiscal authority to manipulate the next-period election probability and policies in order to maximize their plurality in the current election. The political uncertainty, combined with the conflict over the allocation of public budget, creates incentives to act strategically.

We can now define a politico-economic equilibrium that takes into account voting decisions.

**Definition 3 (Markov-Perfect Equilibrium)** Given the initial condition \( b_0 \in [\bar{b}, \tilde{b}] \), a Markov-perfect equilibrium is an equilibrium feasible allocation such that, for each \( t \geq 0 \) and for each \( j, j^+ \in \{ \mathcal{L}, \mathcal{R} \} \), the differentiable policy rules \( \mathcal{T}^j : [\bar{b}, \tilde{b}] \rightarrow [0, \bar{\tau}] \), \( \mathcal{I}^j : [\bar{b}, \tilde{b}] \rightarrow [0, \bar{\iota}] \), \( \mathcal{G}^j : [\bar{b}, \tilde{b}] \rightarrow [0, \bar{g}] \), \( \mathcal{B}^j : [\bar{b}, \tilde{b}] \rightarrow [\bar{b}, \tilde{b}] \), and election probability \( \mathcal{P}^{j^+} : [\bar{b}, \tilde{b}] \times \{ \mathcal{L}, \mathcal{R} \} \rightarrow [0, 1] \) satisfy the following points:

i. The policy rules \( \tau_t^j = \mathcal{T}^j (b_t) \), \( \iota_t^j = \mathcal{I}^j (b_t) \), \( g_t^j = \mathcal{G}^j (b_t) \), and \( b_{t+1}^j = \mathcal{B}^j (b_t) \) are equal to the arg max of Eqs. (14) and (15) for party \( \mathcal{L} \) and \( \mathcal{R} \), respectively, subject to the budget constraint

\[
TR \left( \mathcal{T}^j (b_t) \right) + \rho h \left( \mathcal{I}^j (b_t) \right) \mathcal{B}^j (b_t) \geq \mathcal{G}^j (b_t) + \mathcal{I}^j (b_t) + b_t
\]

and the transition law \( h_{t+1} = \mathcal{H} (\mathcal{I}^j (b_t)) h_t \).
ii. The election probability \( p_j^t = \mathcal{P}_j \) satisfies the functional equation (16).

The first point requires the political control variables to be chosen to maximize the party’s objective function, constrained to fiscal feasibility requirements, the policy rule of the next period and the human capital technology. The second point requires that, if the equilibrium exists, it must satisfy the fixed-point requirement – i.e., Eq. (16). For notational purposes, let denote \( \pi_1 \equiv \frac{\chi}{\phi} (\lambda^a \phi^a \theta^a + \lambda^o \phi^o \theta^o) \) and \( \pi_2 \equiv \frac{\chi}{\phi} \lambda^a \phi^a \beta \theta^o \) the weights attached to the marginal benefits of public goods by the current living voters and the next-period elderly agents as internalized by the current adults, respectively, \( \pi_3 \equiv \frac{\chi}{\phi} (\beta, R) \lambda^a \phi^a \) the weight attached to the marginal cost of paying tax by the adults, and \( \pi_4 \equiv \frac{\gamma}{\phi} \) the degree of competitiveness in the electoral competition, which is maximum when \( \pi_4 = 0 \). By solving the maximization program of parties with respect to policies, we obtain the following first order derivatives:\(^8\)

\[
\tau_t^j : 0 = \pi_3 \alpha^j u_c^j c_t^j + \zeta TR_{\tau_t^j} \\
g_t^j : 0 = \gamma^j \varphi_t^j + \pi_1 \alpha^j v_{g_t^j} - \zeta \\
u_t^j : 0 = \gamma^j \varphi_t^j + \pi_2 \alpha^j E_j \left[ v_{g_t^j+1} G(b_{t+1}) \right] H_{\tau_t^j} - \zeta \left( 1 - \rho H_{\tau_t^j} b_{t+1} \right) \\
b_{t+1}^j : 0 = \pi_2 \alpha^j \frac{\partial \mathcal{P}_j (b_{t+1})}{\partial b_{t+1}^j} \left( v(G^L (b_{t+1}) H (i_t^j)) - v(G^R (b_{t+1}) H (i_t^j)) \right) \\
+ \pi_2 \alpha^j E_j \left[ v_{g_t^j+1} \frac{\partial G(b_{t+1}^j)}{\partial b_{t+1}^j} \right] H (i_t^j) + \zeta \rho H (i_t^j)
\]

where \( \zeta \) is the Lagrangian multiplier associated to the government budget constraint.

---

\(^8\)We use the following notation:

\[
E_j \left[ v_{g_t^j+1} \frac{\partial G(b_{t+1}^j)}{\partial b_{t+1}^j} \right] = p_{t+1}^j v_{g_t^j+1} \frac{\partial G(b_{t+1}^j)}{\partial b_{t+1}^j} + \left( 1 - p_{t+1}^j \right) v_{g_t^j+1} \frac{\partial G^{-j}(b_{t+1}^j)}{\partial b_{t+1}^j}
\]

and

\[
E_j \left[ v_{g_t^j+1} G(b_{t+1}^j) \right] = p_{t+1}^j v_{g_t^j+1} G^j (b_{t+1}^j) + \left( 1 - p_{t+1}^j \right) v_{g_t^j+1} G^{-j} (b_{t+1}^j)
\]
5.3 Discussion

The Markov-perfect equilibrium of the voting game is characterized by a set of wedges, which are obtained by rearranging the above derivatives and eliminating the multiplier.\(^9\) Combining Eqs. (18) and (19) yields:

\[
\pi_1 \nu_{gt} + \pi_4 \varphi_{gt} = \pi_3 u_{ct} MCPF_t
\]  

(22)

Condition (22) describes the *intra-temporal wedge of private and public consumption*. It outlines the trade-off between the marginal cost of taxation, due to a reduction in private consumption suffered by the adult cohort, and the marginal benefits of public good provision as internalized by both voters and parties. An increase in \(g_t\) has a double-side benefit. First, it has a positive impact on the utility of both generations. Note that the public consumption increases if the preference weight on public goods is larger (i.e., larger \(\theta^o\)), the power held by the elderly voters is stronger (i.e., higher \(\phi^o\)) and the the turnout rate at election of elderly voters is higher (i.e., higher \(\lambda^o\)). Second, the public good provision increases the parties’ utilities. Thus, the lower the weight attached to the parties office-motives (i.e., lower \(\alpha^j\)), the large are the benefits from the provision of public good. Note that generally the benefits from public good provision received by each generation as the outcome of the electoral competition are larger than the benefits generated by the benevolent government allocation, as described in Eq. (10).

\[
\pi_2 \mathcal{H}_{it} + \pi_4 \frac{\varphi_{it} - \varphi_{gt}(1 - \rho \mathcal{H}_{it} b_{t+1})}{\mathbb{E}[v_{gt+1}, G(b_{t+1})]} = \pi_1 \frac{v_{gt}}{\mathbb{E}[v_{gt+1}, G(b_{t+1})]} (1 - \rho \mathcal{H}_{it} b_{t+1})
\]

(23)

Condition (23) is obtained combining Eqs. (19) and (20) and captures the *intertemporal strategic wedge between growth and unproductive spending*. The intuition behind this result is best conveyed discussing its parts in isolation. First, suppose that parties are purely office seeking, i.e., \(\gamma^j = 0\), and the government has not access to the market of bonds. In this context, the parties are homogeneous and converge to the same fiscal platform. Thus, Eq. (23) reduces

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\(^9\)The superscript \(j\) is disregarded for expositional purposes.
to $H_{t_1}v_{t+1} = \frac{\pi_1 + \pi_{2_t}}{\pi_2 v_{t+1}}$. Given any concave $v(\cdot)$, the larger the public investment, the higher is the provision of public good. Therefore, the two arms of the intergenerational spending are strategic complements. Such an effect hinges on both the adult’s taste for the next-period public good as quantified by $\pi_2$ and by the political influence exercised by elderly voters, $\phi^p$. The intuition is the following: adult voters support productive policies, as they are entitled to democratically grab a share of the next-period production by exerting their political power when old.\(^{10}\) If governments can raise revenue by borrowing risk-free one-period bonds, then investing in productive assets generates an additional benefit. By increasing the future labor productivity, the current investment enlarges the government possibility to repay debt in the next-period as captured by the term $(1 - \rho H_{t_1}b_{t+1})$. Finally, let us introduce political frictions as measured by diverging preferences of parties for public investment and unproductive spending. For party $L$ this extra term provides an additional motive to implement growth-oriented policies, while the opposite holds for party $R$. However, the pro-redistributive party also recognizes the positive impact of productive investment in expanding the size of government, as measured by $\varphi_{g_t}(1 - \rho H_{t_1}b_{t+1})$. Compared to the social planner decision of productive investments, Eq. (11), policymakers do not necessarily internalize the long-lasting positive impact of public investments. In contrast, they evaluate how productive policies will affect the government borrowing conditions, which, in turn, will influence future fiscal programs, as described here below.

Finally, plugging Eq. (19) and (21), we obtain the \textit{intertemporal strategic wedge between public debt and unproductive spending}

\[
E \left[ v_{g_{t+1}} \frac{\partial G(b_{t+1})}{\partial b_{t+1}} \right] = -\frac{\rho}{\pi_2} \left( \pi_1 v_{g_t} + \pi_4 \varphi_{g_t} \right) + \left( \frac{v(G_{L}(b_{t+1})\mathcal{H}(t_t)) - v(G_{R}(b_{t+1})\mathcal{H}(t_t))}{\mathcal{H}(t_t)} \right) \frac{\partial P(b_{t+1})}{\partial b_{t+1}}
\]

\(^{10}\)See Lancia and Russo (2013) for a detailed discussion on the strategic role of human capital in an overlapping generation model with probabilistic voting.
Following the argument in the previous paragraph, suppose that political decisions are independent of partisan motivations. In this context, the intertemporal wedge boils down to:

\[
\frac{\partial G(b_{t+1})}{\partial b_{t+1}} = -\rho \frac{\pi_1}{\pi_2} \frac{v_{nt}}{v_{nt+1}}.
\]

As a result, the current public debt crowds out the next-period provision of public good. Furthermore, the larger the weight on the next-period public good (i.e., larger \(\pi_2\)), the stronger is the disciplining effect exercised by the adult voters, who anticipate that increasing debt will prompt a fiscal adjustment, which will reduce their future public good consumption.\(^{11}\)

By adding the political friction, the disciplining effects might be further enhanced because of two reasons. First, more a party cares at the margin of public good provision, the larger are the expected losses due the government shrinking as a consequence of debt repayment. Second, when parties are heterogeneous, then the probability of being reelected might be an endogenous function of the state variable. Precisely, if \(\frac{\partial P(b_{t+1})}{\partial b_{t+1}} < 0\), then the disciplining effects are further magnified. The intuition is simple: if the manipulation of the economic activity is considered harmful, as it decreases the future provision of public good, then forward-looking voters may punish such policies at the polls. The party marginal loss would be proportional to the next-period old swing voter ”entity”, namely to the size of:

\[
v(G^L(b_{t+1})H(t_t)) - v(G^R(b_{t+1})H(t_t)).
\]

On the contrary, if \(\frac{\partial P(b_{t+1})}{\partial b_{t+1}} > 0\), then it would implied that voters are rewarding politicians who deliver high spending and high debt. However, this scenario can be reasonably sustained only if high debt crowds in productive investments, which, in turn, sustain more generous public spending. The specific functional form of \(P(\cdot)\) depends upon the parametric assumptions. Unlike the social planner, parties without commitment use debt in a strategic fashion. The internalization of the strategic political feedback on future policies and election probability affects the current political decisions. When such endogenous effects are disciplining, political competition rules out fiscal immiseration possibilities, which where instead present in the optimal allocation, as reported in Eq. (12).

In the following Section, we provide an analytical characterization in a simple economy.

\(^{11}\)See Song et. al. (2012) for a discussion on the strategic role of debt in an overlapping generation model with probabilistic voting.
6 Illustrative Example

In this Section, we provide an analytical characterization of the politico-economic equilibrium under the assumptions that agents’ labor supply is inelastic and intergenerational externalities of human capital are absent. In particular, we set \( l(\tau_t) = 1 \) – implying that \( \bar{\tau} = 1 \) – and \( \vartheta = 1 \).

The Markov-perfect equilibrium is obtained as the limit of a finite-horizon equilibrium whose characteristics do not significantly depend on the time horizon, as long as it is long enough. We parameterize (i) the preferences over public and private consumption as the logarithmic type, \( u(c) = \log(c) \) and \( v(g) = \log(g) \), thus \( \xi(\beta, R) = 1 + \beta \); and (ii) the partisan component of parties’ utility as \( \mu_j \log \iota_t + \eta_j \log g_t \), where \( \mu^L > \mu^R \) and \( \eta^L < \eta^R \) if parties disagree on the composition of public spending, or, alternatively, \( \mu^j = \eta^j = \gamma^j \) with \( \gamma^L > \gamma^R \) if parties disagree on the size of government. Then, the following Proposition holds.

**Proposition 1 (Equilibrium with inelastic labor)** Given the initial condition \( b_0 \in [b^j, b^{\bar{b}}] \) where

\[
\begin{align*}
\iota_t &= I^j(b_t) = \frac{1}{1-w^Z} \frac{\mu^j + \eta^j + \alpha_j \pi_1(\pi_1 + \pi_2 + \pi_3)}{\mu^j \rho Z} (w - b_t); \\
g_t &= G^j(b_t) = \frac{\eta^j + \alpha_j \pi_1}{\mu^j + \eta^j + \alpha_j \pi_1(\pi_1 + \pi_2 + \pi_3)} (w - b_t); \\
b^j_{t+1} &= B^j(b_t) = \frac{\mu^j w Z^2 - \alpha_j \pi_3 (1 - w^Z \rho) \mu^j}{\mu^j \rho Z}; \\
h_{t+1} &= H^j(b_t) = \frac{Z}{1 - w^Z} \frac{\mu^j}{\mu^j + \eta^j + \alpha_j (\pi_1 + \pi_2 + \pi_3)} (w - b_t); \\
t^j_t &= \rho H^j(b_t) b^j_{t+1} = g^j_t + \iota^j_t + b_t.
\end{align*}
\]

Furthermore, the equilibrium probability to be elected is constant, i.e., \( \frac{\partial P^j - (b_t)}{\partial b_t} = 0 \).

**Proof.** (See appendix).  

The Markov-perfect equilibrium delivers policy rules that are linear function of the asset variable. Note that \( \rho \) must be sufficiently low to guarantee existence. If the price of bond were
too high, then the cost of indebtedness would be too low. This would lead to an excessive amount of government bond holding and to the violation of the commitment of repayment of public debt, as \( w < b_t \) for each \( t \). The debt policy is not affected by the current level of public debt. Therefore, the economy converges to the balanced growth after one period. In equilibrium, governments may accumulate assets if \( w \rho Z < \frac{\alpha j \pi_3}{\mu j + \alpha j \pi_3} < 1 \), which is easier satisfied if party \( \mathcal{R} \) is the incumbent. Remarkably, the economy does not revert to immiseration regime, as it happened in the absence of political frictions\(^\text{12}\). The election probability, although endogenously determined, has mute strategic effects (to be described later).

### 6.1 Comparative Statics

Let consider the scenario of parties’ disagreement over the composition of government spending. Without loss of generality, we adopt the convenient parameter restrictions \( \alpha^j = 1 - \eta^j - \mu^j \) with \( \alpha^\mathcal{L} = \alpha^\mathcal{R} = \alpha, \mu^\mathcal{L} = \mu^\mathcal{R} + \Delta \), and \( \eta^\mathcal{L} = \eta^\mathcal{R} - \Delta \). These conditions imply that the two parties attach an equal weight to the office-seeking motive and introduces symmetry in the equilibrium strategy. It is easy to show that in transition, for any feasible initial condition \( b_0 \), the amount of productive investment implemented by the left-wing party is larger than the amount implemented by the right-wing party because of the extra reasons incorporated in the parties’ policy-motive, i.e., \( \mu^\mathcal{L} > \mu^\mathcal{R} \). For a symmetric argument, the public good provision delivered by the rightist politics is more generous compared to the public good provided by the leftist governments, as \( \eta^\mathcal{L} < \eta^\mathcal{R} \). Due to the parameters’ restriction, the tax rate implemented in transition is the same for both parties. However, in steady state the following Corollary holds.

**Corollary 1 (Long-run)** \( i^\mathcal{L*} = i^\mathcal{R*}, b^\mathcal{L*} > b^\mathcal{R*}, g^\mathcal{L*} < g^\mathcal{R*}, \text{ and } \tau^\mathcal{L*} > \tau^\mathcal{R*} \).

**Proof.** (See appendix). \( \blacksquare \)

Remarkably, although party \( \mathcal{L} \) cares more of the investment in productive assets compared

\(^{12}\)Like Song et. al., (2012), the absence of political frictions drives our economy to immiseration, that is maximum debt, fiscal expropriation and no provision of public good. Conversely, in the Markov-perfect equilibrium with political frictions, the commitment of parties to either provide more generous public good or to implement larger public investment averts immiseration, without requiring additional frictions in the labor market.
to party $\mathcal{R}$, in steady state the two parties converge to the same level of growth. This is because the sustenance of larger investments reduces the cost of debt and leads to a weaker fiscal discipline, namely to a stronger indebtedness. The high level of debt crowds out both the next-period investments and the public good in per-efficiency units. In addition, the undisciplined fiscal policy induces in the long-run to a heavier fiscal burden – compared to the case with $\mathcal{R}$ as incumbent party – in order to guarantee repayment of public debt. These findings are supported by the empirical evidences, so as reported in the Introduction. Differences in partisanship do not have a significant effect on growth, whereas they partially explain differences in countries’ debt accumulation, because of the parties’ strategic use of debt, as introduced by Persson and Svensson (1989) and Alesina and Tabellini (1990).

Our framework is quite flexible to evaluate how exogenous changes in parameters affect the equilibrium election probability. Although constant over time, the election prospects internalizes all the strategic incentives embedded in the dynamic voting game, as stated in the following Corollary.

**Corollary 2 (Election Prospects)** When $\phi^o$ is sufficiently large, the election probability of party $\mathcal{L}$, $\mathcal{P}^j - (b)$, is increasing in $\lambda^o$. Moreover, it is increasing in $\alpha^\mathcal{L}$ and $\psi$, and decreasing in all the other parameters.

**Proof.** (See appendix).

Not surprisingly, larger the incumbency bias, the higher is the probability for the incumbent party to be reelected. Moreover, by definition, larger the weight that the party attaches to her office-motives, stronger is her ability to paddle their influence among voters and, in turn, larger is the probability of winning the election. There is a third force, which works in favor of party $\mathcal{L}$ being elected, that is the adults’ turn-out rate at election, $\lambda^o$. In the arena of inter-generational conflicts, adult voters have stronger interest in supporting productive investment compared to the old voters, since such investments generate more generous next-period aggregate spending in public good. When the elderly single-mindedness is particularly strong, i.e.,
\(\phi\) is sufficiently large, such conflict is even more exacerbated, and adults have wider interests in supporting party \(\mathcal{L}\) compared to party \(\mathcal{R}\). However, as pointed out in Corollary 1, political support to party \(\mathcal{L}\) will lead to a deteriorate welfare state in the long-run. Therefore, surprisingly, are the younger agents, the voters that – by supporting the pro-growth party – act in a more short-sighted fashion. The negative consequences related to unbalanced budgets will be subsequently borne by future generations, which the current generation is not caring of because of the absence of altruism. Clearly, due to the restrictive parametric assumptions, the closed form solution does not permit to investigate more complex dynamic feedbacks. For example, in a more general environment election probabilities would be a non trivial function of the state variable. In this context, we may explore the endogenous effect of growth and debt on the incumbent’s prospect of reelection. If manipulating economic activity is considered harmful, then forward-looking voters should punish such policies at the polls, instead of reward them. As a consequence, the probability of being re-elected would be decreasing in the amount of public debt, partially reestablishing fiscal discipline in the political competition.

**Corollary 3 (Political Competition)** *Political competition promotes growth and reduces public debt.*

**Proof.** (See appendix).

The present papers gives a clear cut on the equilibrium relation between political competition, growth and public debt. For both fiscal dimensions, political competition is a discipline device. The reason is straightforward. In this model, a good proxy of political competition is the weight the politicians attach to pure office-seeking motives. Indeed, in the extreme case of \(\alpha^j = 1\), parties converge to the mean voter political platform and win the election with ex-ante probability one half. The internalization of the expected utility of adult voters plays a crucial role to politically discipline parties. To reduce the negative impact of high debt on public good provision, parties reduce the debt burden of future generations and enforce a proactive growth to finance a larger size of government.
**Corollary 4 (Political Polarization)** Political polarization leads to lower growth and higher public debt.

Political polarization is here measured by the parameter $\Delta$. When $\Delta$ approaches zero, parties’ polarization vanishes. The two parties, although policy motivated, have not conflicting policy interests. Therefore, policymakers converge to an identical platform that maximizes the utility of the adult and elderly swing voters subject to common policy restrictions. Then, lower political polarization implies a more intense political competition. As a consequence, the implications of a different degree of party polarization on public debt and growth turn out to be specular to the effects induces by a change in the intensity of political competition, as stated in Corollary 4.

Parties may also disagree on the size of government, without disagreeing on the composition of public spending, namely suppose that $\mu^L = \eta^L = \gamma^L$ with $\gamma^L > \gamma^R$. Party $L$ is now better addressed as pro-state party, whereas the party $R$ is considered by voters as supporting pro-market positions. In this scenario, the following Corollary holds.

**Corollary 5 (Disagreement on the Size of Government)** In the long-run, the pro-market government generates higher growth, lower public debt, and larger provision of public good.

**Proof.** (See appendix).

The party that attaches more weight to the public spending is the less disciplined policymaker, as it drives the economy towards larger debt and lower growth. Although the commitment on public good induces fiscal discipline, such effect is always perfectly compensated by the less disciplining behavior induced by the commitment on public investment, if the weight attached to the two fiscal dimensions is the same, as it was here.

### 7 Conclusion

In this paper, we propose a novel mechanism that identifies equilibrium strategic relations between political frictions, growth, and public debt. The main result is that public investments
boost the public debt; whereas, unproductive spending disciplines governments indebtedness. Furthermore, we support the idea that stronger political competition leads to higher growth and lower public debt.

Our model is simple and can be generalized in a number of directions. First, we have restricted attention to commitment of debt repayment. Second, we have abstracted from old-targeted transfers to treat physical capital as a not payoff-relevant variable. Nevertheless, we believe that a future extension of the model should discuss default as well as pension benefits.
8 Appendix

Proposition 1. The resolution strategy involves two steps. First, starting from a sufficiently large and finite date $T$, we compute the first-order conditions for fiscal policies, and we solve by backward induction for each time $T - t$ with $t = 0, 1, ..., T$, subject to (i) the Euler condition for saving and leisure, (ii) the fiscal feasibility conditions, and (iii) the policy rules of future periods. Second, we recursively determine the conditions for the existence of the fixed points.

At the terminal date $T$, voters have a one-period temporal horizon. Hence, both parties set $b_{T+1} = 0$. Furthermore, constituencies values the promised political platforms on the basis of the sole current impact. Thus, the swing voters in the adult and elderly cohort are equal to:

$$\tilde{\sigma}^a_T = \log (w - g^L_T - \iota^L_T - b_T) - \log (w - g^R_T - \iota^R_T - b_T) + \theta^a (\log g^L_T - \log g^R_T)$$  \hspace{1cm} (25)

and

$$\tilde{\sigma}^o_T = \theta^o (\log g^L_T - \log g^R_T)$$  \hspace{1cm} (26)

Let solve the fiscal problem of party $j$ when the incumbent is $j_-$ with $j, j_- \in \{L, R\}$. The objective function is equal to $\mu^j \log \iota^j_T + \eta^j \log g^j_T + \alpha^j p^j_-(b_T)$, where $p^j_-(b_T) = \frac{1}{2} - \psi^j_- + \frac{\chi}{\phi} (\lambda^a \phi^a \tilde{\sigma}^a_T + \lambda^o \phi^o \tilde{\sigma}^o_T)$ is the share of voters belonging to the adult and elderly cohort that supports party $L$ when the incumbent is $j_-$. Recall that $\psi^j_- = \psi$ if the incumbent $j_-$ is the party $R$, while it is $\psi^j_- = -\psi$ if the incumbent $j_-$ is the party $L$. The first-order conditions for $\iota^j_T$ and $g^j_T$ are, respectively, $rac{\iota^j_T}{w - b_T - \iota^j_T} = \frac{\mu^j}{\mu^j + \eta^j \lambda^a \phi^a + \pi_1}$ and $\frac{g^j_T}{w - b_T - \iota^j_T} = \frac{\eta^j + \alpha^j \pi_1}{\eta^j + \alpha^j \lambda^a \phi^a + \alpha^j \pi_1}$ where $\pi_1 = \frac{\chi}{\phi} (\lambda^a \phi^a \theta^a + \lambda^o \phi^o \theta^o)$. Solving for fiscal policies yields:

$$\iota^j_T = \frac{\mu^j}{\mu^j + \eta^j + \alpha^j \left(\frac{\chi}{\phi} \lambda^a \phi^a + \pi_1 \right)} (w - b_T)$$

and

$$g^j_T = \frac{\eta^j + \alpha^j \pi_1}{\mu^j + \eta^j + \alpha^j \left(\frac{\chi}{\phi} \lambda^a \phi^a + \pi_1 \right)} (w - b_T)$$
Plugging the equilibrium policy rules for parties $\mathcal{L}$ and $\mathcal{R}$ at period $T$ into Eqs. (25) and (26) yields the equilibrium swing voters:

$$
\tilde{\sigma}_T^a = \log \frac{\alpha^c}{\alpha^R} + (1 + \theta^a) \log \frac{\mu^R + \eta^R + \alpha^R \left( \frac{x}{\phi} \lambda^a \phi^a + \pi_1 \right)}{\mu^c + \eta^c + \alpha^c \left( \frac{x}{\phi} \lambda^a \phi^a + \pi_1 \right)} + \theta^a \log \frac{\eta^c + \alpha^c \pi_1}{\eta^R + \alpha^R \pi_1}
$$

and

$$
\tilde{\sigma}_T^o = \theta^o \log \frac{\eta^c + \alpha^c \pi_1}{\eta^R + \alpha^R \pi_1} \left( \frac{\mu^R + \eta^R + \alpha^R \left( \frac{x}{\phi} \lambda^a \phi^a + \pi_1 \right)}{\mu^c + \eta^c + \alpha^c \left( \frac{x}{\phi} \lambda^a \phi^a + \pi_1 \right)} \right)
$$

Given the incumbent $j_-$, the share of population voting for party $\mathcal{L}$ is equal to:

$$
p_{j-} (b_T) = \frac{1}{2} - \psi_j - \frac{X}{\phi} \lambda^a \phi^a \log \frac{\alpha^c}{\alpha^R} + \left( \frac{X}{\phi} \lambda^a \phi^a + \pi_1 \right) \log \frac{\mu^R + \eta^R + \alpha^R \left( \frac{x}{\phi} \lambda^a \phi^a + \pi_1 \right)}{\mu^c + \eta^c + \alpha^c \left( \frac{x}{\phi} \lambda^a \phi^a + \pi_1 \right)} + \pi_1 \log \frac{\eta^c + \alpha^c \pi_1}{\eta^R + \alpha^R \pi_1}
$$

Likewise, the the share of population voting for party $\mathcal{R}$ is $1 - p_{j-} (b_T)$. Hence, $p^c (b_T) - p^R (b_T) = 2\psi$.

At time $T - 1$, the agents born at $T - 2$ have two periods lifespan. Hence, internalizing the policy rules of period $T$, the swing voters in the adult and elderly cohort are, respectively, equal to:

$$
\tilde{\sigma}_{T-1}^a = (1 + \beta) \log \left( w - b_{T-1} - g_{T-1}^c - h \left( \nu_{T-1}^c \right) \right) + \rho h \left( \nu_{T-1}^c \right) \left( b_{T-1}^c \right)
$$

$$
- (1 + \beta) \log \left( w - b_{T-1} - g_{T-1}^R - h \left( \nu_{T-1}^R \right) \right) + \rho h \left( \nu_{T-1}^R \right) \left( b_{T-1}^R \right)
$$

$$
+ \theta^a \left( \log g_{T-1}^c - \log g_{T-1}^R \right) + 2\psi \beta^o \log \frac{\eta^c + \alpha^c \pi_1}{\eta^R + \alpha^R \pi_1} \left( \frac{\mu^R + \eta^R + \alpha^R \left( \frac{x}{\phi} \lambda^a \phi^a + \pi_1 \right)}{\mu^c + \eta^c + \alpha^c \left( \frac{x}{\phi} \lambda^a \phi^a + \pi_1 \right)} \right)
$$

$$
+ \beta \theta^a \left( \left( \log \left( w - b_{T-1}^c \right) - \log \left( w - b_{T-1}^R \right) \right) + h \left( \nu_{T-1}^c \right) - h \left( \nu_{T-1}^R \right) \right)
$$

and

$$
\tilde{\sigma}_{T-1}^o = \theta^o \left( \log g_{T-1}^c - \log g_{T-1}^R \right)
$$

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The objective function of party \( j \) is equal to \( \mu^j \log \nu_{T-1}^j + \eta^j \log g_{T-1}^j + \alpha^j p^j \left( b_{T-1} \right) \), where \( p^j \left( b_{T-1} \right) = \frac{1}{2} - \psi^j + \frac{\chi}{\phi} \left( \chi^a \phi^a \tilde{\sigma}_{T-1}^a + \chi^o \phi^o \tilde{\sigma}_{T-1}^o \right) \). Let denote \( \pi_2 = \frac{\chi}{\phi} \lambda^a \phi^a \beta \theta^o \) and \( \pi_3 = \frac{\chi}{\phi} \left( 1 + \beta \right) \lambda^o \phi^o \). The first-order conditions with respect to \( \nu_{T-1}^j \), \( g_{T-1}^j \), and \( b_T^j \) are, respectively, equal to:

\[
\frac{\nu_{T-1}^j}{w - b_{T-1} - g_{T-1}^j} = \frac{1}{1 - \rho Z b_T^j} \left( \frac{\mu^j + \alpha^j \pi_2}{\mu^j + \alpha^j \left( \pi_2 + \pi_3 \right)} \right)
\]

and

\[
\frac{g_{T-1}^j}{w - b_{T-1} - \nu_{T-1}^j + \rho Z \nu_{T-1} b_T^j} = \frac{\eta^j + \alpha^j \pi_1}{\eta^j + \alpha^j \left( \pi_1 + \pi_3 \right)}
\]

and

\[
b_T^j = \frac{w \left( 1 + \beta \right)}{1 + \beta \left( 1 + \theta^o \right)} - \frac{\beta \theta^o}{1 + \beta \left( 1 + \theta^o \right)} \left( \frac{w - b_{T-1} - g_{T-1}^j - \nu_{T-1}^j}{\rho Z \nu_{T-1}^j} \right)
\]

Solving the system for the fiscal policies yields:

\[
\nu_{T-1}^j = \frac{1}{1 - w \rho Z} \left( \frac{\mu^j}{\mu^j + \eta^j + \alpha^j \left( \pi_1 + \pi_2 + \pi_3 \right)} \right) (w - b_{T-1})
\]

and

\[
b_T^j = \frac{\mu^j w Z \rho - \alpha^j \pi_2 (1 - w \rho Z)}{\mu^j \rho Z}
\]

and

\[
g_{T-1}^j = \frac{\eta^j + \alpha^j \pi_1}{\mu^j + \eta^j + \alpha^j \left( \pi_1 + \pi_2 + \pi_3 \right)} (w - b_{T-1})
\]

Plugging the equilibrium policy rules for parties \( L \) and \( R \) at period \( T - 1 \) into Eqs. (27) and (28) yield the equilibrium swing voters of the adult and elderly cohorts:

\[
\tilde{\sigma}_{T-1}^a = \left( (1 + \beta) + \beta \theta^o \right) \log \left( \frac{\alpha^L}{\alpha^R} \right) + 2 \psi^a \beta \theta^o \log \left( \frac{\eta^L}{\eta^R} \right) \left( \frac{\mu^L + \eta^L + \alpha^L \left( \frac{\chi}{\phi} \lambda^a \phi^a + \pi_1 \right)}{\mu^L + \eta^L + \alpha^L \left( \frac{\chi}{\phi} \lambda^a \phi^a + \pi_1 \right) + \theta^o \log \left( \frac{\eta^L + \alpha^L \pi_1}{\eta^R + \alpha^R \pi_1} \right)} \right)
\]

\[
+ \left( 1 + \beta + \theta^o + \beta \theta^o \right) \log \left( \frac{\mu^R + \eta^R + \alpha^R \left( \pi_1 + \pi_2 + \pi_3 \right)}{\mu^L + \eta^L + \alpha^L \left( \pi_1 + \pi_2 + \pi_3 \right) + \theta^o \log \left( \frac{\eta^L + \alpha^L \pi_1}{\eta^R + \alpha^R \pi_1} \right)} \right)
\]
We conclude the analysis by studying the dynamics of our economy. The transition laws of the share of population voting for party $\mathcal{L}$ when the incumbent is $j_-$ is equal to:

$$p_{j_-}^{T_1} = \frac{1}{2} - \psi_{j_-} + 2\psi_{T_2} \log \frac{\eta^L + \alpha^L \pi_1}{\eta^R + \alpha^R \pi_1} \left( \frac{\mu^R + \eta^R + \alpha^R (\pi_1 + \pi_2 + \pi_3)}{\mu^L + \eta^L + \alpha^L (\pi_1 + \pi_2 + \pi_3)} \right)$$

It follows that the share of population voting for party $\mathcal{R}$ when the incumbent is $j_-$ is equal to:

$$p_{j_-}^{T_2} = \theta^o \log \mu^R + \eta^R + \alpha^R (\pi_1 + \pi_2 + \pi_3)$$

Likewise, the share of population voting for party $\mathcal{R}$ in the swing voters in both cohorts is as follows:

$$\tilde{\sigma}_{j_-}^{T_1} = \theta^o \log \frac{\eta^L + \alpha^L \pi_1}{\eta^R + \alpha^R \pi_1} \left( \frac{\mu^R + \eta^R + \alpha^R (\pi_1 + \pi_2 + \pi_3)}{\mu^L + \eta^L + \alpha^L (\pi_1 + \pi_2 + \pi_3)} \right)$$

At period $T - 2$, the swing voters in both cohorts are as follows:

$$\tilde{\sigma}_{j_-}^{T_2} = \theta^o \log \frac{\eta^L + \alpha^L \pi_1}{\eta^R + \alpha^R \pi_1} \left( \frac{\mu^R + \eta^R + \alpha^R (\pi_1 + \pi_2 + \pi_3)}{\mu^L + \eta^L + \alpha^L (\pi_1 + \pi_2 + \pi_3)} \right)$$

Thus, the equilibrium probability is equal to:

$$p_{j_-}^{T_2} = \frac{1}{2} - \psi_{j_-} + 2\psi_{T_2} \log \frac{\eta^L + \alpha^L \pi_1}{\eta^R + \alpha^R \pi_1} \left( \frac{\mu^R + \eta^R + \alpha^R (\pi_1 + \pi_2 + \pi_3)}{\mu^L + \eta^L + \alpha^L (\pi_1 + \pi_2 + \pi_3)} \right)$$

We conclude the analysis by studying the dynamics of our economy. The transition laws of the
human capital is:

\[ h_{t+1} = \frac{Z}{1 - w\rho Z} \left( \frac{\mu^j}{\mu^j + \eta^j + \alpha^j (\pi_1 + \pi_2 + \pi_3)} \right) (w - b_t) h_t \]

Thus, in balanced growth, we obtain:

\[ b^j_* = \frac{\mu^j wZ - \alpha^j \pi_2 (1 - w\rho Z)}{\mu^j \rho Z} \] (29)

and

\[ \nu^j_* = \frac{1}{\rho Z} \left( \frac{\alpha^j \pi_2}{\mu^j + \eta^j + \alpha^j (\pi_1 + \pi_2 + \pi_3)} \right) \]

and

\[ g^j_* = \frac{\eta^j + \alpha^j \pi_1}{\mu^j + \eta^j + \alpha^j (\pi_1 + \pi_2 + \pi_3)} \left( \frac{\alpha^j \pi_2 (1 - w\rho Z)}{\mu^j \rho Z} \right) \]

and

\[ h_i(v^j_*) = \frac{1}{\rho} \left( \frac{\alpha^j \pi_2}{\mu^j + \eta^j + \alpha^j (\pi_1 + \pi_2 + \pi_3)} \right) \] (30)

\[ QED \]

\[ \text{Corollary 1. First, let consider } b^j_* \text{, as reported in Eq. (29). Since } \mu^R < \mu^C \text{ and } 1 - w\rho Z > 0, \text{ then } b^{C*} > b^{R*}. \text{ By contradiction, suppose the contrary holds, then } \mu^R \alpha^C > \mu^C \alpha^R. \text{ As } \alpha^C = \alpha^R = \alpha, \text{ then this relation cannot be satisfied. The steady state investment implemented by the two parties under the restriction } \alpha^C = \alpha^R = \alpha \text{ is } i^j_* = \frac{1}{\rho Z} \frac{\alpha \pi_2}{1 - \alpha + \alpha (\pi_2 + \pi_3) + \alpha \pi_1}, \text{ for each } j. \]

Third, the steady state provision of public good is:

\[ g^j_* = \frac{\eta^j + \alpha \pi_1}{1 - \alpha + \alpha (\pi_2 + \pi_3) + \alpha \pi_1} \left( \frac{\alpha \pi_2 (1 - w\rho Z)}{\mu \rho Z} \right) \]

for each \( j \). For contradiction, suppose \( g^{C*} > g^{R*} \) then \( \frac{\eta^C + \alpha \pi_1}{\mu^C} > \frac{\eta^R + \alpha \pi_1}{\mu^R} \). Since, \( \mu^R < \mu^C \) and \( \eta^C < \eta^R \), the inequality cannot be satisfied. Finally, the income tax rate implemented by the two parties is \( \tau^j_* \propto -\frac{1 - \alpha - \eta^j + \alpha \pi_2}{\mu^j}. \) Suppose \( \tau^C_* < \tau^{R*} \). Then, we would require
\[ 1 - \alpha (1 - \pi_3) < \frac{\mu^R \eta^c - \mu^L \eta^R}{\mu^L - \mu^c}. \] Since \[ \frac{\mu^R \eta^c - \mu^L \eta^R}{\mu^L - \mu^c} = 1 - \alpha, \] then the inequality cannot be verified. \(\text{QED} \]

**Corollary 2.** If \(\alpha^c = \alpha^R = \alpha = 1 - \mu^j - \eta^j\), then Eq. (16) simplifies to \(p^j - \frac{1}{2} - \psi^j + (\pi_1 + 2\psi\pi_2) \log \frac{\eta^c + \alpha_1}{\eta^c + \alpha_2}. \) It is straightforward to show that the derivatives with respect to \(\psi\) and \(\alpha\) are positive, whereas the derivative with respect to \(\lambda^a\) is equal to:

\[
\frac{\partial p^j}{\partial \lambda^a} = (2\beta \psi \phi^a \theta^a + (\phi^a \theta^a - \phi^o \theta^o)) \log \frac{\eta^c + \alpha_1}{\eta^c + \alpha_1} \\
- (\phi^a \theta^a - \phi^o \theta^o) \frac{\alpha_1 (\eta^c - \eta^R)(\pi_1 + 2\psi\pi_2)}{(\eta^R + \alpha_1)(\eta^c + \alpha_1)}
\]

If \(\phi^o\) is sufficiently large, then \(\frac{\partial p^j}{\partial \lambda^a} > 0. \) \(\text{QED} \]

**Corollary 3.** Let evaluate the growth rate and debt level when varying the parameter \(\alpha^j\). Then, the following inequalities hold: \(\frac{\partial b^j}{\partial \alpha^j} = -\frac{\pi_2 (1 - w_\rho Z)}{\rho \rho Z} < 0 \) and \(\frac{\partial h^j}{\partial \alpha^j} \propto \pi_2 (\mu^j + \eta^j) > 0. \) \(\text{QED} \]

**Corollary 5.** Let \(\mu^j = \eta^j = \gamma^j\) with \(\gamma^c > \gamma^R\). From the equilibrium policies in Proposition 1, it directly follows that party \(L\) supports more public investment, more public good and a higher level of public debt. In steady state, the larger indebtedness reduces the economic growth, i.e. \(\frac{\partial \lambda^j}{\partial \gamma^j} = \frac{\partial}{\partial \gamma^j} \left[ \frac{1}{\rho} \frac{\alpha_1 \pi_2}{2 \gamma^j + \alpha_2 (\pi_1 + \pi_2 + \pi_3)} \right] < 0, \) and the provision of public good, i.e. \(\frac{\partial \gamma^j}{\partial \gamma^j} = \frac{\partial}{\partial \gamma^j} \left[ \frac{\gamma^j + \alpha_2 \pi_1}{2 \gamma^j + \alpha_2 (\pi_1 + \pi_2 + \pi_3)} \left( \frac{\alpha_2 \pi_2 (1 - w_\rho Z)}{\gamma^j \rho Z} \right) \right] < 0. \) \(\text{QED} \]
References


