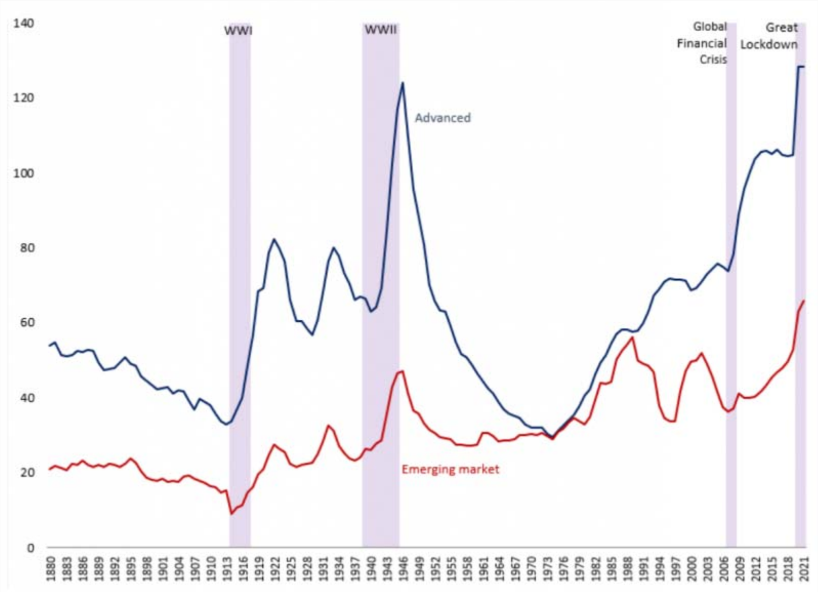
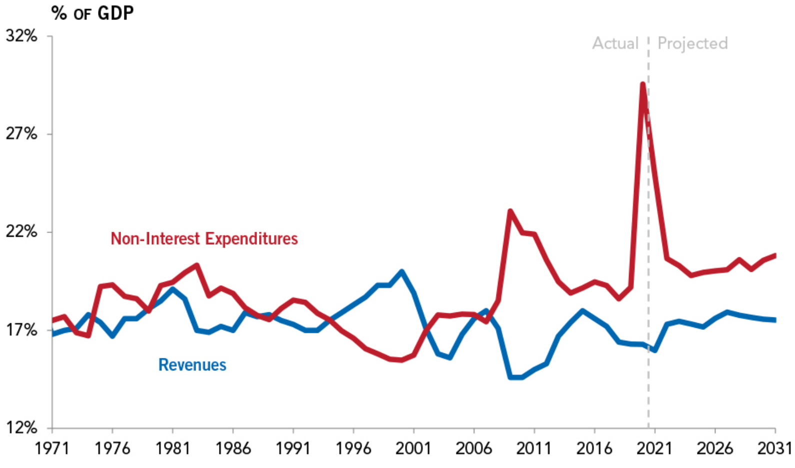
Debt Sustainability

Debt Dynamics in the World

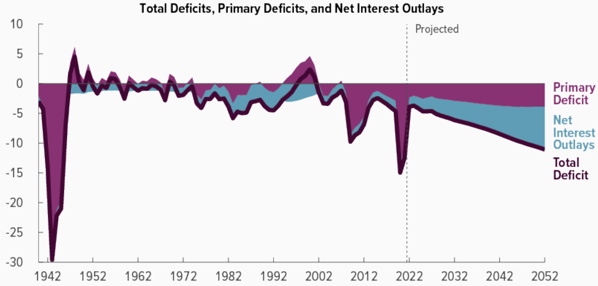


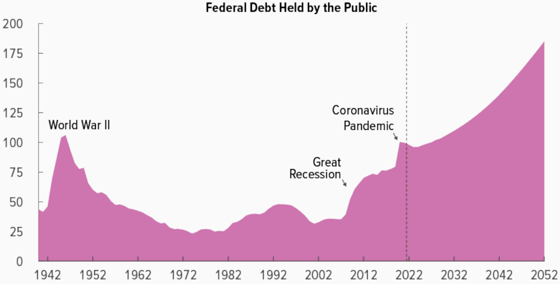
US Dynamics of Exp. and Rev.



SOURCES: Congressional Budget Office, *The Budget and Economic Outlook: 2021 to 2031*, February 2021; and Office of Management and Budget, *Historical Tables, Budget of the United States Government: Fiscal Year 2021*, February 2020.

US Dynamics of Primary Surplus





US Debt Dynamics and Debt Ceiling

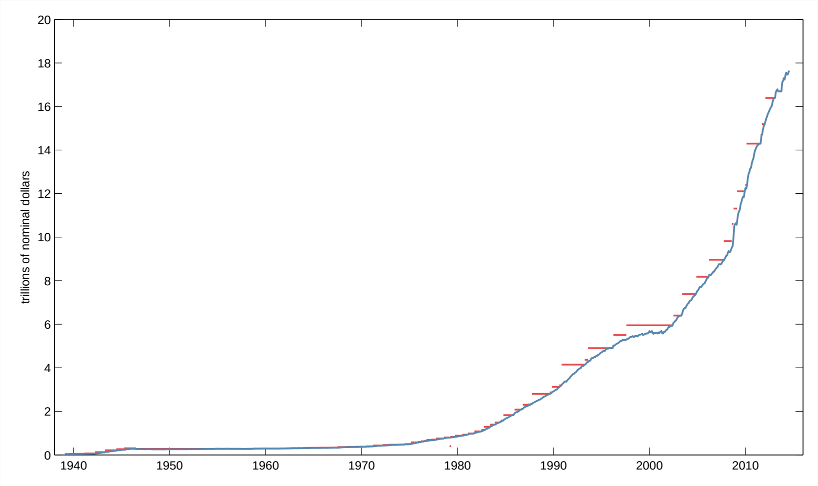


Figure 1: Total Debt and Limit: Nominal, 1939-2014, Hall and Sargent 2015

Relevant Questions

- What is a **sustainable** government debt-to-GDP ratio?
- How much **should** future primary surplus be raised in order to finance the $34.5 trillion dollar debt outstanding as of Q1 2024 in the US?
- Is the projected dynamics of primary surpluses consistent with such an optimal surplus path?

1. Debt Arithmetics

Debt Arithmetics

- Government's budget constraint is "*the least controversial equation in macroeconomics*" (Hall and Sargent, 2011)

$$\underbrace{B_t}_{\text{Outstanding Debt}} = \underbrace{X_t}_{\text{Primary Surplus}} + \underbrace{\sum_{s_{t+1}} q(s_{t+1}|s^t)B(s_{t+1}|s^t)}_{\text{Bond Revenue}}$$

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- We consider a more general stochastic setting where bonds are one-period state-contingent (Arrow-Debreu allocations)
  - they can be implemented with noncontingent debt of different maturity (Angeletos, 2002)
- For each state s_{t+1}, the promised payoffs for this state constitutes the new level of government liabilities, B_{t+1}

Debt Arithmetics

- The market value of outstanding debt equals the PDV of current and future primary surpluses + trasv. condition:

$$B_t = \underbrace{\sum_{k\geq 0}\sum_{s_{t+k}} \mathcal{Q}(s_{t+k}|s^t)X(s^{t+k})}_{\text{PDV Surpluses}} + \underbrace{\lim_{k\to\infty}\sum_{s_{t+k}} \mathcal{Q}(s_{t+k}|s^t)B(s_{t+k}|s^t)}_{\text{Trasversality Cond.}}$$

with $\mathcal{Q}(s_{t+k}|s^t) = \prod_{j=1}^{k} q(s_{t+j}|s^{t+j-1})$

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- Debt valuation depends on:

  i How future primary surpluses are discounted

  → **Properties of** $\mathcal{Q}(\cdot)$

  ii How primary surplus responds to shocks and debt

  → **Properties of** $X(\cdot)$

Debt Sustainability

- Key question in the sustainability context is whether the path of government debt has to satisfy a set of transversality conditions
- Since government borrowing is limited by the government's ability to lend, the answer will depend on the assumptions about the market
- Sustainability is inherently a general equilibrium issue

Debt Arithmetic: Properties of Q

- Consider a Lucas (1978) exchange economy with IL agents:

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) U(C^i(s^t))$$

- Agents are endowed with stochastic Y_t^i used for consumption or sold to Gvt

$$C_t^i \leq Y_t^i - T_t^i - \sum_{s_{t+1}} q(s_{t+1}|s^t) A^i(s_{t+1}|s^t) + A_t^i$$

- Asset market clearing condition (zero net supply)

$$\sum_i A^i(s^t) = B(s^t)$$

- Good market clearing condition:

$$\sum_i C^i(s^t) + G(s^t) = \sum_i Y^i(s^t)$$

Debt Arithmetic: Properties of Q

- From the state-by-state FOC for optimality:

$$q(s_{t+1}|s^t) = \pi(s_{t+1}|s^t) \overbrace{\underbrace{\beta \frac{U'(C^i(s_{t+1}|s^t))}{U'(C^i(s^t))}}_{\text{Stochastic Discount Factor (SDF)}}}^{:=M(s_{t+1}|s^t)}$$

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- The t price of claim that pays at $t+k$ is $\mathcal{Q}(s_{t+k}|s^t)$ equal to

$$\pi(s_{t+1}|s^t)\beta\frac{U'(C^i(s_{t+1}|s^t))}{U'(C^i(s^t))} \cdot \pi(s_{t+2}|s^{t+1})\beta\frac{U'(C^i(s_{t+2}|s^{t+1}))}{U'(C^i(s_{t+1}|s^t))} \cdot$$
$$....\pi(s_{t+k}|s^{t+k-1})\beta\frac{U'(C^i(s_{t+k}|s^{t+k-1}))}{U'(C^i(s_{t+k-1}|s^{t+k-2}))}$$
$$= \pi(s_{t+k}|s^t) \cdot M(s_{t+k}|s^t)$$

Debt Arithmetic: Properties of Q

- The intertemporal budget constraint can be rewritten as:

$$B_t = \underbrace{\sum_{k=0}^{T} \mathbb{E}_t[M_{t,t+k} \cdot X_{t+k}]}_{\text{Exp. risk-adjust. PDV of surpluses}} + \lim_{T\to\infty} \mathbb{E}_t[M_{t,t+T} \cdot X_{t+T}]$$

- Suppose for now, trasversality cond. is satisfied

$$B_t = \sum_{k=0}^{\infty} (\mathbb{E}_t[M_{t,t+k}] \cdot \mathbb{E}_t[X_{t+k}] + COV_t[M_{t,t+k}, X_{t+k}])$$

- The risk-free interest rate is:

$$1/(1 + r^f_{t,k}) = 1/R^f_{t,k} = \mathbb{E}_t[M_{t,t+k}]$$

- The corresponding yield is:

$$y_{t,k} = \frac{1}{k} \log(R^f_{t,k})$$

Debt Arithmetic: Properties of Q

- Note that although bonds are risk free, the risk-free rate may vary over time and across states of nature of the economy
- The level of sustainable debt is not simply the sum of expected surplus discounted at the risk-free rate, there is also a covariance term if surpluses correlate with marginal utilities!
- There is a (multiplicative) risk premium on bonds quantified by the excess returns of the asset

$$\mathrm{MRP}_t = \frac{\bar{R}_t^{\text{bond}} - \bar{R}_t^f}{\bar{R}_t^f}$$

$$\text{with} \quad R_t^{\text{bond}} := \frac{B_{t+1}}{\sum q_{t+1} B_{t+1}}$$

Debt Sustainability without Uncertainty

- The traditional approach of debt sustainability (flow approach) focuses only on the trajectory of debt-to-gdp ratio (Ball, Elmendorf, and Mankiw, 1998; Barrett, 2018; Blanchard, 2019)
- This approach recognizes that debt dynamics depend on the difference between the interest on government debt r and the growth rate of the economy γ
- It does not explicitly consider conditions under which forward-looking investors are willingly hold public debt
- Within our framework, this approach is valid only in deterministic setting or if risk-neutral lenders (external lenders)

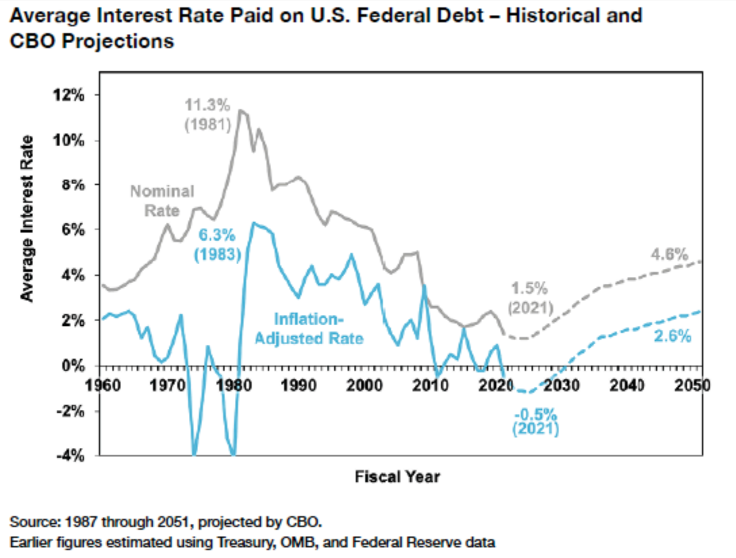
Debt Sustainability without Uncertainty

- In a deterministic setting or when lender is risk neutral, the intertemporal budget simplies to:

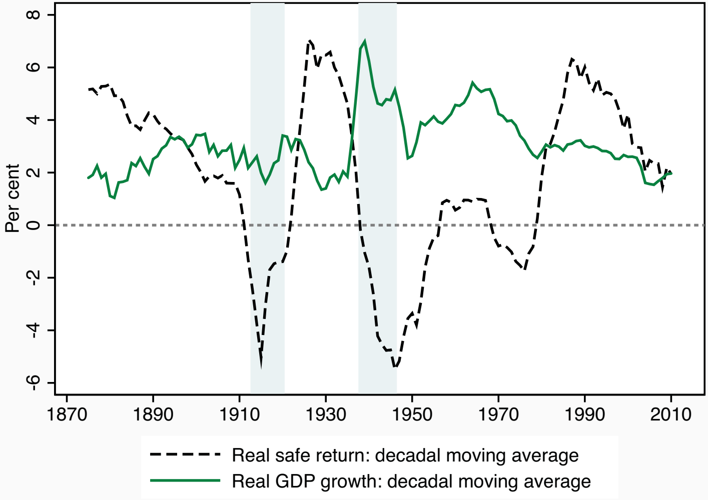
$$\frac{B_t}{Y_t} = \underbrace{\sum_{k\geq 0} \left(\frac{1+\gamma}{1+r}\right)^k \frac{X_{t+k}}{Y_{t+k}}}_{\text{PDV} \quad \text{Surpluses}} + \underbrace{\lim_{k\to\infty} \left(\frac{1+\gamma}{1+r}\right)^k \frac{B_{t+k}}{Y_{t+k}}}_{\text{Trasversality} \quad \text{Cond.}}$$

- where the gross growth rate includes also population growth $1+\gamma$=(1+g)(1+n)
- The COV term is zero and there is not risk premium on bonds
- Debt sustainability implications depends on $r-\gamma$

US Dynamics of Exp. and Rev.



$r - \gamma$



Behavior of $r - \gamma$, Jorda, Schularick, and Taylor (2016)

Dynamic Efficiency

- If $r > \gamma$ (dynamic efficiency cond in deterministic setting), then the intertemporal budget constraint is well defined since:

$$\lim_{k\to\infty} \left(\frac{1+\gamma}{1+r}\right)^k \frac{B_{t+k}}{Y_{t+k}} = 0$$

- Debt increases over time and reaches a maximum sustainable:

$$\bar{b} = \frac{1+r}{r-\gamma}\bar{x}$$

$$\text{with} \quad b = B/Y, x = X/Y$$

- Debt sustainability requires necessarily to raise positive surpluses in the long run

Dynamic Inefficiency

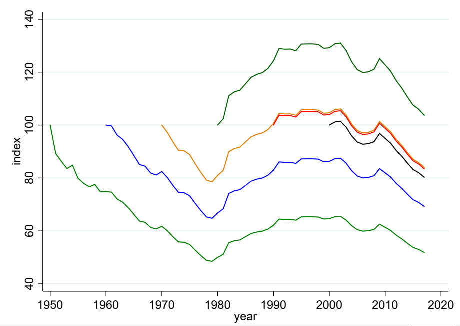
- If $r < \gamma$ (dynamic inefficiency cond in deterministic setting), then the intertemporal budget constraint is NOT well defined since:

$$\lim_{k \to \infty} \left(\frac{1+\gamma}{1+r} \right)^k \frac{B_{t+k}}{Y_{t+k}} \to \infty$$

- There is infinite rollover of debt: debt is never repaid or a Ponzi scheme exists
- Although debt is increasing, the ratio over GDP decreases even if taxes are not raised: unlimited fiscal space
- Assumption: Level of debt does not affect demand of bond and risk premium, but unexpected changes in $r - \gamma$ have strong implication for debt sustainability

Blanchard, 2019

Figure 5: Debt dynamics, with zero primary balance, starting in year t, using the non-tax adjusted rate



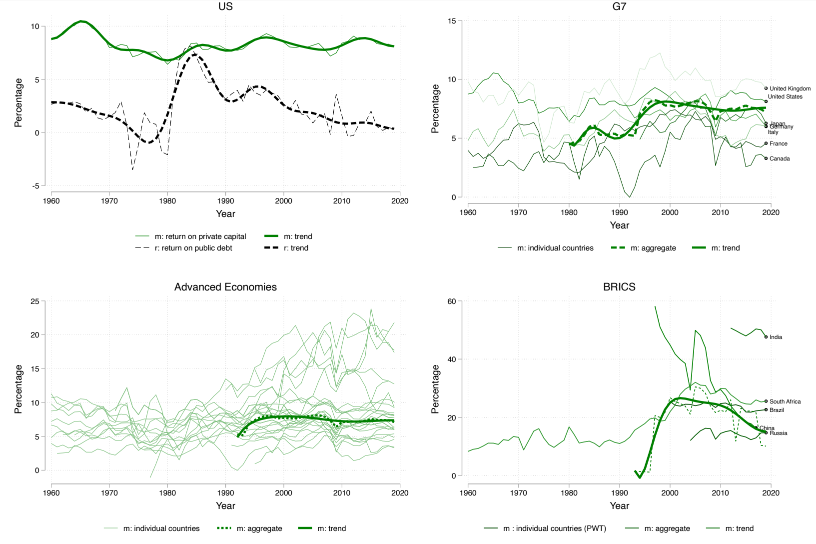
Blanchard, 2019

"My purpose . . . is not to argue for more public debt. It is to have a richer discussion of the costs of debt" (Blanchard 2019)

$r - \gamma$ or $m - \gamma$? (Reis, 2021)

- Which r should we use?
- Average returns on US government debt are low. But average returns on equity m, and measures of the marginal product of capital are comfortably higher than γ
- In a frictionless world without uncertainty all interest rates are the same
- Since there are many different rate of returns (Jorda, et al, 2019), this fact tells us we are in a world with uncertainty and frictions
- The value of debt is finite:
  - The observed finite value of debt needs to be interpreted with a sensible PDV formula

$r-\gamma$ or $m-\gamma$? (Reis, 2022)



Reis, 2022

$r - \gamma$ or $m - \gamma$? (Reis, 2022)

- The return on private investment (marginal product of capital) m can be used as alternative discount factor
- m is the opportunity cost of investors of holding public debt rather than private capital
- We observe $m > r$ in the data and $m > g$, hence using m as discount factor allows to solve for trasver. cond.
- Why investors buy gvt bonds if $m > r$?
  - $\Rightarrow$ An explanation: Public debt carries a liquidity/service value (Krishnamurthy and Vissing-Jorgensen 2012)

$r - \gamma$ or $m - \gamma$? (Reis, 2022)

- The intertemporal budget is now

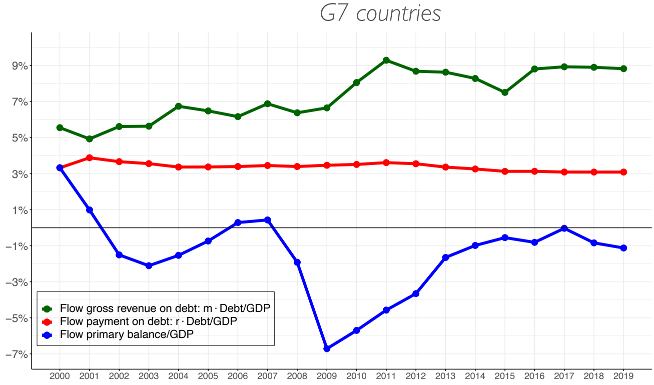
$$\frac{B_t}{Y_t} = \underbrace{\sum_{k\geq 0} \left(\frac{1+\gamma}{1+m}\right)^k \frac{X_{t+k}}{Y_{t+k}}}_{\text{PDV Surpluses}}$$

$$+ \underbrace{\sum_{k\geq 1} \left(\frac{1+\gamma}{1+m}\right)^k \frac{(m-r)^k D_{t+k}}{Y_{t+k}}}_{\text{Debt Revenue}}$$

$$+ \underbrace{\lim_{k\to\infty} \left(\frac{1+\gamma}{1+m}\right)^k \frac{B_{t+k}}{Y_{t+k}}}_{\text{Trasversality Cond.}}$$

- Debt is repaid (trasversality cond. satisfied) and Gvt receives extra revenue related to debt service value

How Much is Debt Revenue



m-r = 4% seems quite good, and so debt revenue of 75-100% of GDP.
But could be twice as high, or closer to zero.

Reis, 2022

Debt Sustainability Under Uncertainty

- Under uncertainty, using safe real return of bonds r as a discount rate is incorrect
- Future risky flows must be evaluated by the lenders' marginal utility or pricing kernel (Bohn, 1995; 1998; 2005)

Debt Sustainability Under Uncertainty

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- In an **ILRA model**, the pricing kernel for claims on $t+k$

$$\mathcal{Q}(s_{t+k}|s^t) = \pi(s_{t+k}|s^t)\overbrace{M(s_{t+k}|s^t)}^{\text{SDF}}$$

$$= \pi(s_{t+k}|s^t)\beta^k \frac{u'(C(s_{t+k}|s^t))}{u'(C(s^t))} = \pi(s_{t+k}|s^t)\beta^k \frac{u'(Y_{t+k} - G_{t+k})}{u'(Y_t - G_t)}$$

- The SDF is thus counter-cyclical!

Debt Sustainability Under Uncertainty

- The SDF depends on aggregate risk (shocks to growth rate)
- If **surpluses are correlated with SDF**

$$B_t = \sum_{k=0}^{T} \sum_{s_{t+k}} \mathcal{Q}(s_{t+k}|s^t) X(s^{t+k}) + \lim_{T\to\infty} \sum_{s_{t+T}} \mathcal{Q}(s_{t+T}|s^t) X(s^{t+T})$$

$$= \sum_{k=0}^{T} \Big\{ \mathbb{E}_t[M(s_{t+k}|s^t)]\mathbb{E}_t[X(s^{t+k})]$$
$$+ COV_t \Big[M(s_{t+k}|s^t), X(s^{t+k})) \Big] \Big\} + \lim_{T\to\infty} \mathbb{E}_t[M(s_{t+T}|s^t) \cdot X(s^{t+T})]$$

Bohn (1995) Example

- Under uncertainty, trasversality cond. can be satisfied even if $r^f < \bar{\gamma}$ where r^f is the risk free rate and $\bar{\gamma}$ is mean growth rate
- Consider stochastic growth $1 + \gamma_t = \frac{Y_t}{Y_{t-1}}$ with γ_t iid and $\mathbb{E}[\gamma_t] = \bar{\gamma}$
- Fixed spending over GDP: $g = \frac{G_t}{Y_t}$ and thus $c = \frac{C_t}{Y_t}$
- Coefficient of risk aversion $\gamma \geq 1$
- The risk-free rate is:

$$\frac{1}{1 + r^f} = \beta \mathbb{E}_t \Big[\Big(\frac{C_{t+1}}{C_t} \Big)^{-\alpha} \Big] = \beta \mathbb{E}_t \Big[\Big(\frac{cY_{t+1}}{cY_t} \Big)^{-\alpha} \Big] = \beta (1 + \bar{\gamma})^{-\alpha}$$

- Larger β, $\bar{\gamma}$ or lower α implies lower r^f

Bohn (1995) Example

- Consider a debt-stabilizing tax policy that allows to keep debt over GDP constant $\frac{B_{t+1}}{Y_{t+1}} = d$ at the end of the period
- A constant debt-GDP ratio requires primary surpluses to be **negatively correlated with GDP growth**:

$$\begin{aligned} X_t = T_t - g \cdot Y_t &= (1 + r^f) B_{t-1} - B_t \\ &= (1 + r^f) \cdot d \cdot Y_{t-1} - d \cdot Y_t \\ &= d \cdot Y_{t-1} \cdot (r^f - \gamma_t) \end{aligned}$$

- What is relevant for sustainability is not r^f but the return on GDP-indexed claims:

$$\frac{1 + \bar{\gamma}}{1 + R} \equiv \mathbb{E}_t \Big[\beta \Big(\frac{Y_{t+1}}{Y_t} \Big)^{-\alpha} \frac{Y_{t+1}}{Y_t} \Big]$$

Bohn (1995) Example

- The intertemporal budget constraint is:

$$B_t = \sum_{k=0}^{T} \left\{ \mathbb{E}_t \left[\left(\frac{1}{1+r^f} \right)^k \mathbb{E}_t \left[X_{t+k} \right] + \underbrace{COV_t \left(\beta^k \left(\frac{Y_{t+k}}{Y_t} \right)^{-\alpha}, X_{t+k} \right)}_{\text{positive}} \right\} + \lim_{T\to\infty} \mathbb{E}_{t+T} \left[\beta^T \left(\frac{Y_{t+T}}{Y_t} \right)^{-\alpha} \cdot B_{t+T} \right]$$

- where the transversality condition can be written as:

$$\lim_{T\to\infty} d\mathbb{E}_{t+T} \left[\beta^T \left(\frac{Y_{t+T}}{Y_t} \right)^{-\alpha} \cdot (1+\gamma_{t+T}) \right] Y_t = \lim_{T\to\infty} d \left(\frac{1+\bar{\gamma}}{1+R} \right)^T Y_t = 0 \quad \text{if} \quad R > \bar{\gamma}$$

- In this case positive debt may be sustained even if $E_t[X_{t+k}]$ is perpetually negative!

Bohn (1995) Example

- The example of Bohn (1995) is illustrative not realistic, as in reality primary surplus is pro-cyclical
- It illustrates the role of debt management and of asset pricing considerations
- To satisfy trasversality conditions notion of dynamic efficiency ($r^f > \gamma$) is neither necessary nor sufficient (Kocherlakota, 2023; Bloise and Reichlin, 2023)
  - The relevant $r - \gamma$ comparison uses the yield on long-term bond
- It is crucial to specify properties of X including its state-contingent specification

Properties of X: Fiscal Reaction Function

- How to test if an observed policy is on a trajectory consistent with the intertemporal budget and No-Ponzi condition?
- Older literature (Hamilton and Flavin, 1986; Trehan and Walsh, 1988) computes panel unit-root tests and cointegration tests
- Bohn (1998) shows that these test are inconclusive: Difficult to reject a unit root in debt-GDP ratio, but changes in debt may be due to shocks or corrective measures?
- Surplus depends on fiscal rules: The **fiscal reaction function (FRF)** (or policy feedback rule) describes how surplus responds to debt

Properties of X: Fiscal Reaction Function

- (Weak) sufficient condition for sustainability is that surplus **positively** reacts to increase of outstanding debt (Bohn,1998; 2008)

  $$x_t = \rho b_t + \mu_t$$

  with $x_t = X_t/Y_t$, $b_t = B_t/Y_t$, with $\rho >$ and $\mu_t = \alpha Z_t + \epsilon_t$ bounded composite of other factors (war time spending) and an error term

- if $\rho > r$, the fiscal policy follows a debt stabilizing rule (mean reversion)

- Bohn shows that in the period $1793 - 2003$ both surplus and debt as share of GDP are stationary time series and thus ignoring a stationary determinants of surplus as Z_t would lead to omitted bias.

Properties of X: Fiscal Reaction Function

Model:	**Main Model**	With debt-squared	With time trend	With AR(1) for outlays	With surplus ex. seignorage
Coefficient on:	**(1)**	(2)	(3)	(4)	(5)
Initial Debt d_t^*	**0.121**	0.094	0.116	0.117	0.119
	(11.3; 8.6)	(11.6; 8.2)	(10.1; 8.3)	(11.3; 8.6)	(11.3; 8.6)
Constant	**-.030**	-.030	-0.033	0.0003	-.031
	(-10.6; -8.4)	(-10.4; -8.7)	(-8.7;-8.4)	(0.1; 0.2)	(-10.9; -8.7)
Temporary	**0.088**	0.087	0.091	0.085	0.090
output $\tilde{y}_t$	**(2.2; 1.9)**	(2.1; 1.8)	(2.2; 2.0)	(2.1; 1.8)	(2.2; 2.0)
Temporary	**-0.815**	-0.817	-0.823	-0.742	-0.818
outlays $\tilde{g}_t$	**(-19.6;-11.0)**	(-19.6;-11.0)	(-19.5;-11.2)	(-20.0;-11.4)	(-19.9;-11.3)
Squared debt		0.020			
$(d_t^* - \bar{d})^2$		(0.8; 0.8)			
Time Trend			$3*10^{-5}$		
			(1.0;1.45)		
R-squared	0.689	0.691	0.691	0.690	0.697

Notes: Estimates of equation (16) in the text. Entries are coefficient estimates. Entries in brackets are the ordinary t-statistics and heteroskedasticity-consistent t-statistics, respectively. Temporary outlays are computed as different between actual military outlays from their permanent component as implied by an AR(2) process, except in Col.4. Col. 4 uses actual military outlays as regressor, as explained in the text.

Determinants Primary Surplus/GDP, 1973-2003, Bohn (2005)

Fiscal Fatigue

- For debt sustainability it is sufficient that surpluses increase (linearly) with debt, but debt might increase unboundedly
- Large literature attempted to find evidence on the shape of FRF (D'Erasmo et al., 2016)
- Based on cross-country panels evidence of positive response of surplus to debt
- But non-linearities of FRF (**fiscal fatigue**): Primary surpluses raised more slowly to rising debt, which implies a finite debt limit if risk of default is introduced (Ghosh et al., 2013)

Fiscal Fatigue

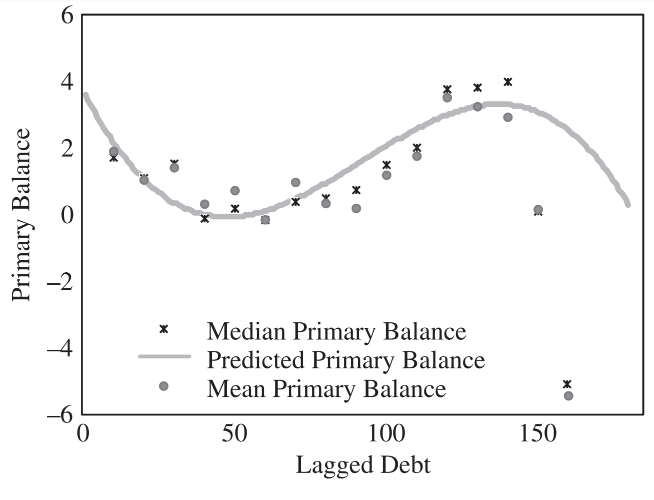
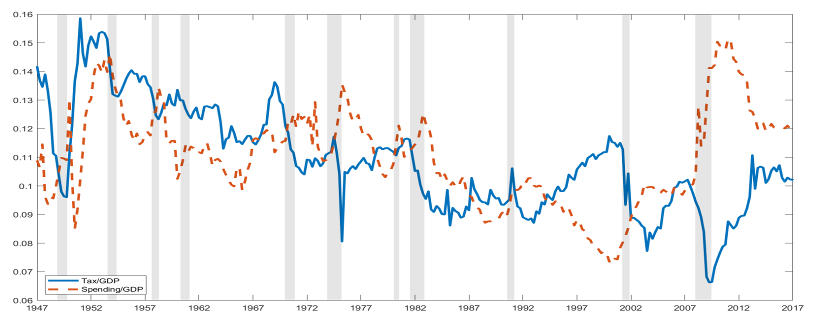


Figure 2: Fiscal Fatigue (Gosh et. al., 2013). Data computed for 23 advanced economies, 1970-2007

Puzzling Facts on Debt Sustainability

Debt Valuation Puzzle



X=T-G is highly procyclical: Surplus claim carries a business cycle risk premium

Debt Valuation Puzzle

- Using the debt arithmetics and assuming ILRA, the presence of priced aggregate risk generates a negative $COV_t[M_{t,t+k}X_{t+k}]$
- Negative covariance lowers the government's fiscal capacity and the value of outstanding debt
- The risk in surpluses creates a large gap between observed value of outstanding debt and the PDV of surpluses
- Vibrant current literature to address this puzzle (Jiang, et. al., 2022; 2023)

Debt Valuation Puzzle

- To quantify $\sum_j \mathbb{E}_t[M_{t,t+j}X_{t+j}]$ need to take a stance on:
  i The time-series properties of revenue and spending X_t
  ii. A stochastic discount factor $M_{t,t+j}$ to discount these cash flows and hence the risk premium
- Jiang et al. consider a VAR where tax and spending policies are allowed to depend on rich set of variables (Cholensky identification)
- Set the risk premium on surplus equal to the risk premium on a GDP claim since both taxes and spending are cointegrated with GDP

Debt Valuation Puzzle

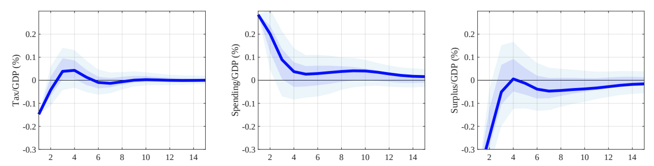
Table 1: State Variables

Position	Variable	Mean	Description	Sample Mean
1	π_t	π_0	Log Inflation	3.16%
2	$y_t^{\$}(1)$	$y_0^{\$}(1)$	Log 1-Year Nominal Yield	4.26%
3	$yspr_t^{\$}$	$yspr_0^{\$}$	Log 5-Year Minus Log 1-Year Nominal Yield Spread	0.58%
4	x_t	x_0	Log Real GDP Growth	2.95%
5	$\Delta \log d_t$	μ_d	Log Stock Dividend-to-GDP Growth	-0.18%
6	$\log d_t$	$\log d_0$	Log Stock Dividend-to-GDP Level	-1.26
7	pd_t^M	pd_0^M	Log Stock Price-to-Dividend Ratio	3.54
8	$\Delta \log \tau_t$	μ_τ	Log Tax Revenue-to-GDP Growth	0.02%
9	$\log \tau_t$	$\log \tau_0$	Log Tax Revenue-to-GDP Level	-1.74
10	$\Delta \log g_t$	μ_g	Log Spending-to-GDP Growth	0.65%
11	$\log g_t$	$\log g_0$	Log Spending-to-GDP Level	-1.75

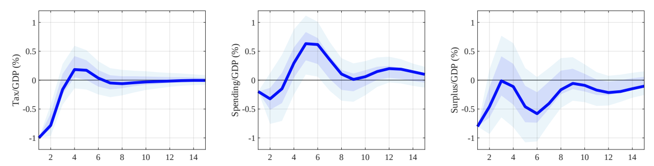
Error Correction Model Under the assumption of cointegration between log tax revenue and log GDP and between log spending and log GDP

Debt Valuation Puzzle

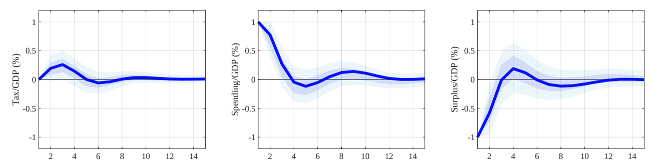
Panel A: −1% Shock to GDP Growth.



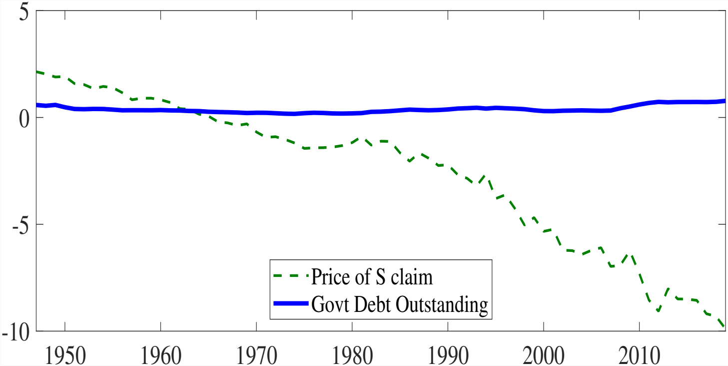
Panel B: −1% Shock to Tax-to-GDP.



Panel C: 1% Shock to Spending-to-GDP.

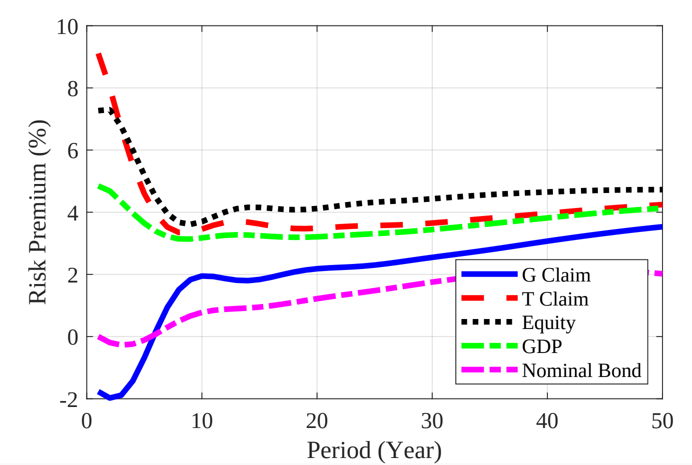


Debt Valuation Puzzle



Debt valuation puzzle (Jiang et. al., 2022): Market value of debt larger than value of surplus claims

Gvt Debt Risk Premium Puzzle

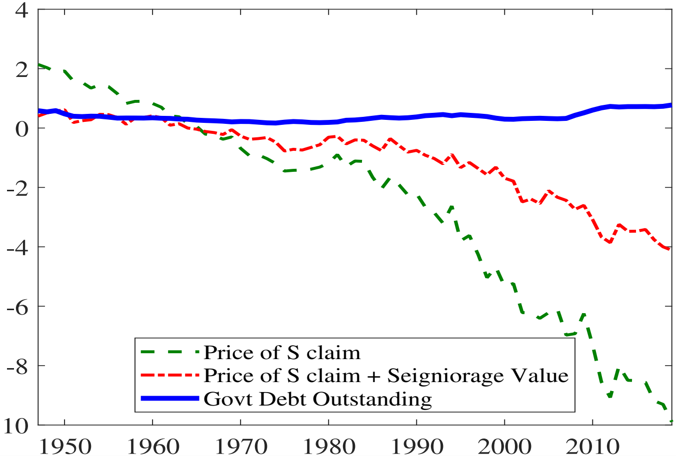


Gvt Debt Risk Premium Puzzle (Jiang et. al., 2022): Yields on treasury bonds are lower than interest rate investors should earn given the surplus risk

How Can the Puzzles Be Explained?

- Bond investors rationally expect major and permanent fiscal correction in the future $\Rightarrow$ Corrections should be of unrealistic magnitude
- Rational bubble in government debt, i.e., trasversality condition not satisfied $\Rightarrow$ In standard ILRA asset pricing model with realistic aggregate risk no bubble; Possible if OLG
- Convenience yields: Treasury bonds pay lower yields than the discount rate implied from SDF $\mathbb{E}[M_{t,t+k}] = \frac{e^{-\lambda_t}}{1+r_t^f}$

Can Convenience Yields Close the Gap?



λ measured as spread between Aaa-rated corporate bonds and Treasury securities against the US government debt-to-GDP ratio

Recap from Debt Sustainability Puzzles

- The sustainable debt should be **smaller** than what observed
- Evidence for the US after WW-II, what about other countries? Chen, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2022) find no evidence of a similar valuation puzzle in the U.S. pre-WW-I or post WW-II in the U.K.
- Current evidence points towards mispricing of the U.S. Treasury portfolio $\Rightarrow$ persistent mispricing when there are limits to arbitrage (Shleifer and Vishny, 1997)
- Nice comment of Gale to Jiang et al (2022): Surpluses are in fact not exogenous, they depend on debt which the gvt aims always to repay $\Rightarrow$ residual claimant of risk are tax payers!

Debt Sustainability

- **Bohn, H.**, "The sustainability of budget deficits in a stochastic economy". Journal of Money, Credit and Banking, 27(1):257–271, 1995.
- **Mehrotra N.**, "Implications of low productivity growth for debt sustainability". Technical report. Brown University, 2018
- **Mehrotra N., Sergeyev D.**, "Debt sustainability in a low interest rate world", Journal of Monetary Economics, 2021, 124,1-18.
- **Ghosh, A., Kim J., Mendoza, E., Ostry, J. D., and Qureshi, M.** Fiscal fatigue, fiscal space and debt sustainability in advanced economies. The Economic Journal, 123(566):F4–F30, 2013.
- **Blanchard, O.**, "Public debt and low interest rates". American Economic Review, 109(4): 1197–1229, 2019.
- **Angeletos, G.-M.**, 2002. "Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure," The Quarterly Journal of Economics, 117(3), 1105-1131.

Relevant Papers

Debt Sustainability

- **Jordà, O., K. Knoll, D. Kuvshinov, M. Schularick, and A. M. Taylor**. 2019. "The Rate of Return on Everything, 1870–2015." Quarterly Journal of Economics, 134(3), 1225-1298. (https:www.macrohistory.netdatabase)
- **Kocherlakota, N.,** 2023. "Public Debt Bubbles In Heterogeneous Agent Models With Tail Risk," International Economic Review
- **Reis, R.**, 2021. "The constraint on public dept when r¡g," BIS Working Papers 939, Bank for International Settlements
- **Cochraine, J.**, 2021. "r¡g," Working Papers - **Brunnermeier, M. K, S. A Merkel, and Y. Sannikov**. 2024. "Debt as Safe Asset." JPE Forthcoming
- **Zhengyang, J., H. Lustig, S. Van Nieuwerburgh, and M. Z Xiaolan**. 2021. "The U.S. Public Debt Valuation Puzzle". Working Paper CEPR

Intergenerational Insurance

Objective

- Use a (stochastic) framework to define optimal surpluses as a function of debt
- Under complete market, optimal consumption is stationary (see Lecture IV) and therefore surpluses do not depend on debt
- Need a friction $\Rightarrow$ Limited enforcement generate endogenous market incompleteness (Alvarez and Jerman, 2000)
- Need an OLG framework to allow to optimally roll-over debt (even without bubble)

A Normative OLG Framework under Frictions

I. **Normative approach**: Policies are optimally chosen by a benevolent planner, who issues bonds in zero net supply and raises taxes, no capital:
   - ◇ as Stokey and Lucas (1983) but without distortionary taxes and in OLG
   - ◇ or as Samuelson (1958) but with shocks

II. **Overlapping generations** model with aggregate (growth) and idiosyncratic **(distributional)** shocks

III. **Limited enforcement** frictions: The planner cannot raise surplus freely but must respect **participation constraints** of each individual

- The model provides new insights on properties of $\mathcal{Q}$ and X

Preview: Properties of Q^*

- In an **OLG model**, the pricing kernel for claims on $t+k$

$$Q^*(s_{t+k}|s^t) = \pi(s_{t+k}|s^t)M^*(s_{t+k}|s^t)$$

$$= \pi(s_{t+k}|s^t)\beta^k \frac{u'(C^o_{t+k})}{u'(C^y_{t+k-1})} \cdot \frac{u'(C^o_{t+k-1})}{u'(C^y_{t+k-2})} \cdot \ldots \cdot \frac{u'(C^o_{t+1})}{u'(C^y_t)}$$

$$= \pi(s_{t+k}|s^t)\beta^k \frac{u'(C^o_{t+k})}{u'(C^y_{t+k})} \cdot \frac{u'(C^y_{t+k})}{u'(C^y_{t+k-1})} \cdot \ldots \cdot \frac{u'(C^o_{t+1})}{u'(C^y_{t+1})} \cdot \frac{u'(C^y_{t+1})}{u'(C^y_t)}$$

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$$= \pi(s_{t+k}|s^t)\beta^k \frac{u'(C^o_{t+k})}{u'(C^y_{t+k})} \cdot \frac{u'(C^y_{t+k})}{u'(C^y_{t+k-1})} \cdot \ldots \cdot \frac{u'(C^o_{t+1})}{u'(C^y_{t+1})} \cdot \frac{u'(C^y_{t+1})}{u'(C^y_t)}$$

- SDF depends also on **how consumption is distributed between generations**, not only on stochastic growth $\Rightarrow$ If fiscal policies cannot insure fully, distributional shocks matter

Preview: Properties of X^*

- The optimal framework allows to derive the optimal FRF $X^*(s^t)$ **endogenously**
- Optimal surpluses are higher when:
  - the outstanding debt is higher
  - growth is higher
  - young is richer than old

Preview: Properties of X^*

- The optimal framework allows to derive the optimal FRF $X^*(s^t)$ **endogenously**
- Optimal surpluses are higher when:
  - the outstanding debt is higher
  - growth is higher
  - young is richer than old
- The FRF must be a **non-linear** function of debt:
  - below $\tilde{b}$, surplus rises linearly with debt
  - above $\tilde{b}$, surplus rises at a decreasing rate when debt increases in order to provide incentives, but at an increasing rate when debt approaches its (endogenous) upper limit to avoid infinite risk premium

Preview: Implications of X^* and Q^*

The SDF varies with both aggregate and distributional risk due to limited enforcement friction:

$$B_t^* = \sum_{k \geq 0} \sum_{s_{t+k}} \mathcal{Q}^*(s_{t+k}|s^t) X^*(s^{t+k})$$

$$= \sum_{k \geq 0} \{\mathbb{E}_t[M^*(s_{t+k}|s^t)]\mathbb{E}_t[X^*(s^{t+k})] + \overbrace{COV_t\left[M^*(s_{t+k}|s^t), X^*(s^{t+k}))\right]}^{\theta^* \lesseqgtr 0}\}$$

where the covariance term is:

$$\theta^* = \overbrace{\theta_z}^{>0 \ \ (cov \ \ from \ \ z)} + \mathbb{E}_t[M^*(z_{t+k}|z^t)]\mathbb{E}_t[X^*(z^{t+k})] \cdot \overbrace{\theta_g}^{<0 \ \ (cov \ \ from \ \ g)}$$

Preview: Implications of X^* and Q^*

- The observed high level of debt can be more sustainable than what (most of the) existing theories demonstrate

- The fiscal rules defining taxes, expenditure and bond issuance should however be designed such that they respect limited enforcement constraints