Risk Sharing Under Limited Enforcement

Risk Sharing Under Limited Enforcement (Kocherlakota,1996; Worrall at al. 1988,1994,2002)

- Consider stochastic model with 2 ex-ante identical risk-averse agents (A and B)

- Total endowment is $1 = y_t^A + y_t^B$, let $y_t^A \equiv y_t$, with random $y_t = \{y(1), y(2), ..., y(N)\}$ such that $\Pr(y_t = y(s)) = \pi(s)$ and feasibility requires $c_t^A + c_t^B = 1$, let $c_t^A \equiv c_t$

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- Problem: Characterize Pareto optimal consumption distribution when transfers among agents are voluntary
- Voluntary means that agents transfer part of their income if it is in their interest $\Rightarrow$ There is not a Gvt that can fully enforce policies

Participation Constraints

- **Participation constraints**: Agents no incentives to walk away (reversion to autarky):

$$u(c(s^t)) + \sum_{j=1}^{\infty} \beta^j \sum_{s_{t+j}} \pi(s_{t+j}|s^t) u(c(s^{t+j}))$$

$$\geq v_s^{aut,A} \equiv u(y(s_t)) + \sum_{j=1}^{\infty} \beta^j \sum_{s_{t+j}} \pi(s_{t+j}|s^t) u(y(s_{t+j}))$$

$$u(1 - c(s^t)) + \sum_{j=1}^{\infty} \beta^j \sum_{s_{t+j}} \pi(s_{t+j}|s^t) u(1 - c(s^{t+j}))$$

$$\geq v_s^{aut,B} \equiv u(1 - y(s_t)) + \sum_{j=1}^{\infty} \beta^j \sum_{s_{t+j}} \pi(s_{t+j}|s^t) u(1 - y(s_{t+j}))$$

- An allocation is sustainable if it satisfies **participation constraints** for any history s^t

Promised Utility Approach

- We want to find the allocation $\{c(s^t)\}$ that maximizes total welfare (exp. sum of utilities) but respect PC and implement it through transfers
- Note that PC are infinite and forward looking $\Rightarrow$ Need a recursive representation
- Use **promise utility approach** (Abreu, Pearce and Stacchetti, 1990), where $\upsilon_t = \mathbb{E}_{t-1} \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$ is used as state variable together with s_t (rather than the history s^t) upon which current consumption and next period promises are defined

Recursive Problem

The Pareto optimal problem in recursive form is:

$$V(\upsilon) = \max_{c_s, \omega_s} \sum_s \pi_s [u(1 - c_s) + \beta V(\omega_s)]$$

s.t.:

$$\begin{aligned}
\text{PK} &: \sum_s \pi_s [u(c_s) + \beta \omega_s] \geq \upsilon \quad (\lambda) \\
\text{PC}^A &: u(c_s) + \beta \omega_s \geq v_s^{aut,A} \quad (\pi_s \mu^A) \\
\text{PC}^B &: u(1 - c_s) + \beta V(\omega_s) \geq v_s^{aut,B} \quad (\pi_s \mu^B) \\
c_s &\in [c_{\min}, c_{\max}] \\
\omega_s &\in [\omega_{\min}, \omega_{\max}]
\end{aligned}$$

where PK is the **promise keeping constraint** and PC^A and PC^B are the **participation constraints** of A and B

First Order Condition

- The FOCs are

$$c_s : -u'(1-c_s) + \lambda u'(c_s) + \mu^A u'(c_s) - \mu^B u'(1-c_s) = 0$$
$$\omega_s : V'(\omega_s) + \lambda + \mu^A + \mu^B V'(\omega_s) = 0$$

- Rearranging we get:

$$\frac{u'(1-c_s)}{u'(c_s)} = \frac{\lambda + \mu^A}{1+\mu^B}, \quad V'(\omega_s) = -\frac{\lambda + \mu^A}{1+\mu^B}$$
$$\Rightarrow \frac{u'(1-c_s)}{u'(c_s)} = -V'(\omega_s)$$

together with the envelope condition

$$V'(v) = -\lambda$$

Optimal Allocation

- In the optimum, either PC^A or PC^B is binding, or both slack
- Case I: If one of the two PC is binding, the optimal (c_s, ω_s) solves:

$$\frac{u'(1-c_s)}{u'(c_s)} = -V'(\omega_s) \Rightarrow c_s = g(\omega_s) \text{ with } g' > 0$$

$$u(c_s) + \beta\omega_s = v_s^{aut,A} \text{ or } u(1-c_s) + \beta V(\omega_s) = v_s^{aut,B}$$

The allocation depends only on current s (amnesia property):

$$\omega_s \equiv \bar{\omega}_s > \upsilon \text{ if } \mu^A > 0 \text{ and } \omega_s \equiv \underline{\omega}_s < \upsilon \text{ if } \mu^B > 0$$

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- Case II: If neither PC binds, then

$$V'(\omega_s) = -\lambda, V'(\upsilon) = -\lambda \Rightarrow \omega_s = \upsilon \text{ and } \frac{u'(1-c_s)}{u'(c_s)} = -V'(\upsilon)$$

- Consumption depends on past history of shocks

Intergenerational Risk Sharing Under Limited Enforcement

Research Questions

I. How should **optimal intergenerational insurance** be structured under **limited enforcement** frictions?

II. What are the implications in terms of **risk-spreading** across generations?

What We Do

- We characterize **optimal intergenerational insurance** under **limited enforcement** when agents belong to finitely-lived **OLG**

... while the past literature has characterized:

i. Optimal intergenerational risk sharing under **full enforcement** (e.g., Aiyagari and Peled, 1991)

ii. Intergenerational risk sharing as a voting equilibrium, which is **not** necessarily optimal (e.g., Cooley and Soares, 1999)

iii. Optimal risk sharing under limited enforcement when agents are **infinitely lived** (e.g., Thomas and Worrall, 1988)

Mechanism in a Nutshell

A trade-off **Incentives** versus **Efficiency**:

✓ **Incentives**: Risk is partially spread onto future generations to provide incentives to the current generation to not walk away

⇒ Consumption depends on past shocks

✓ **Efficiency**: Period *resetting* in the provision of incentives to offset the welfare losses of shocks propagation

⇒ Consumption periodically resets to welfare maximising levels

Policy Implications

- Which policy institution may support the optimal allocation?
- A combination of **taxes**, **transfers** and **state-contingent bonds** can implement the optimal allocation
- Limited enforcement implies a **non-linear fiscal reaction function** to public debt

  $\Rightarrow$ A form of **fiscal fatigue** (Gosh et al 2013)
- Public **debt** as a **safe asset** (good hedge)

  $\Rightarrow$ High sustainable debt even if expected primary surpluses are low (Brunnermeier et al 2022, Jiang et al 2022)

Presentation Outline

i. Model

ii. Benchmark:

  - Full enforcement (First Best)

iii. Optimal Constrained Efficient Allocation

iv. Implications for debt sustainability

Links with the Rest of the Course

We proceed in two steps:

i Characterize the Pareto optimal allocation

ii Implement/Decentralize optimal allocation through fiscal policy instruments

$\Rightarrow$ As in Mirrlees approach (see Lecture 5)

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We allow for nonlinear taxes/transfers+state-contingent debt

- With full enforcement, the optimal allocation is stationary $\Rightarrow$ as in Stokey and Lucas (1983) with state-contingent debt (see Lecture 4)
- With limited enforcement, history of shocks matters $\Rightarrow$ as in Aiyagari et al (2002) even if state-contingent debt (see Lecture 4)
- Limited enforcement generates endogenous market incompleteness (Alvarez and Jermann, 2000)

Model

- Discrete time $t = 0, 1, 2, \ldots, \infty$
- Two-period living agents: Young and Old
- Total endowment $e_t = e_t^y + e_t^o$ of perishable consumption good
- Aggregate (**growth**) risk $\gamma_t = \frac{e_t}{e_{t-1}}$
- Idiosyncratic (**distributional**) risk $s_t = \frac{e_t^y}{e_t} \in \{s(1), s(2)\}$ with $s(1) < s(2)$: When the Young is better off, the Old is worst off
- $\rho_t := (s_t, \gamma_t)$ with probability ϖ and $\rho^t := (\rho_1, \rho_2, ..., \rho_t) \in \mathcal{P}^t$

Model

- Young consumption $C(\rho^t)$ and old consumption $e_t - C(\rho^t)$
- Logarithmic per-period utility function $u(\cdot) = \log(\cdot)$
- Given homogenous utility we can consider the de-trended economy and characterize

$$\{c\} = \{c(\rho^t) = \frac{C(\rho^t)}{e_t} : t \geq 0, \rho^t \in \mathcal{P}^t\}$$

Planner Problem

- The planner chooses $\{c\}$ to maximize the **sum of expected discounted utility** of **all generations**

$$V\left(\{c\};\rho^t\right) = \underbrace{\frac{\beta}{\delta}\left(\log(1-c(\rho^t)\right)}_{\text{Current Old}} + \underbrace{\mathbb{E}_t\left[\sum_{j=t}^{\infty}\delta^{t-j}U\left(\{c\};\rho^j\right)\right]}_{\text{Current Young and Fut. Gen.}}$$

where

$$U\left(\{c\};\rho^t\right) = \log(c(\rho^t)) + \beta\sum_{\rho_{t+1}}\varpi(\cdot,\rho_{t+1})\log(1-c(\cdot,\rho_{t+1}))$$

- **Participation constraints** of the **old**:

$$c(\rho^t) \leq s_t \tag{2}$$

- **Participation constraints** of the **young**:

$$U\left(\{c\};\rho^t\right) \geq \log(s_t) + \beta\sum_{\rho_{t+1}}\varpi(\cdot,\rho_{t+1})\left(\log(1-s_{t+1})\right) \tag{3}$$

Optimal Sustainable Intergenerational Insurance

- An Intergenerational Insurance rule is **sustainable** if the history-dependent consumption plan

$$\{c(\rho^t)\}_{t=0}^{\infty} \in \Lambda := \{\{c(\rho^t)\}_{t=0}^{\infty} \mid \text{(2) and (3)}\}$$

- A Sustainable Intergenerational Insurance rule is **optimal** if it maximizes $V(\{c\}; \rho^t)$ subject to the constraint that the initial old receive a utility of at least ω_0:

$$\log(1 - c(\rho_0)) \geq \omega_0$$

Assumptions

Assumption 1

The idiosyncratic and aggregate shocks are iid: $\varpi(\rho) = \pi(s)\varsigma(\gamma)$

$\Rightarrow$ Any sustainable intergenerational insurance rule $\{c\}$ depends **only** on the history of **idiosyncratic shocks**

Assumption 2

$\beta \sum_r \pi(r) \frac{u'(1-r)}{u'(s)} > 1$

$\Rightarrow$ There **exists** a non-trivial sustainable intergenerational insurance that improves upon autarky

Recursive Formulation

- Let ω_r be the **state-contingent promise** to current young when next-period state is r and the promise to current old is ω
- The planner's optimization problem is:

$$V(s,\omega) = \max_{\{c,(\omega_r)_{r\in\mathcal{I}}\}} \frac{\beta}{\delta}\log(1-c)+\log(c)+\delta\sum\nolimits_r \pi(r)V(r,\omega_r)$$

subject to

$$BC : \omega_{\min}(r) \leq \omega_r \leq \omega_{\max}(r)$$

$$PC_o : c \leq s$$

$$PC_y : \log(c) + \beta\sum\nolimits_r \pi(r)\omega_r \geq \log(s) + \beta\sum\nolimits_r \pi(r)\log(1-r)$$

$$PK : \log(1-c) \geq \omega$$

Notation and Some Simplification

Simplifying assumptions:

- $\beta = \delta$
- BC and PC_o are not binding

Notation:

- μ is the multiplier associated with PC_y
- λ is the multiplier associated with PK
- $c = \mathbf{c}(x)$ and $\omega_r = \mathbf{w}_r(x)$ with $x := (s, \omega)$ are the optimal consumption and state-contingent promised utility

Benchmark: Full Enforcement

Full Enforcement

- The Planner value $V(s, \omega)$ is subject to PK, but not PCy

- The first-order conditions are:

$$\frac{1 - \mathbf{c}(x)}{\mathbf{c}(x)} = 1 + \lambda(x) \quad \text{and} \quad V_\omega(r, \mathbf{w}_r(x)) = 0$$

  The envelope condition is:

$$V_\omega(x) = -\lambda(x)$$

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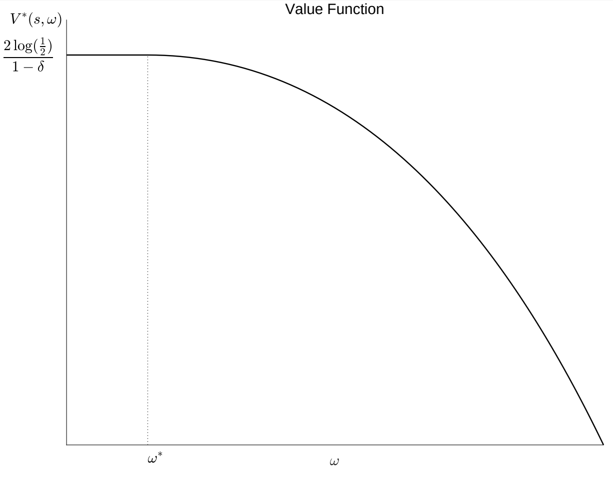
- There is $\omega^* = \log\left(\frac{1}{2}\right) = \sup\{\omega | V_\omega(x) = 0\}$

$\Rightarrow$ If $\omega_0 \leq \omega^*$ then $\lambda = 0$ and $c_0 = c^* := \mathbf{c}(s_0, \omega^*) = \frac{1}{2}$

$\Rightarrow$ If $\omega_0 > \omega^*$ then $\lambda > 0$ and $c_0 = \mathbf{c}(s_0, \omega_0) = 1 - \exp(\omega_0) < \frac{1}{2}$

$\Rightarrow$ $\mathbf{w}_r(s_0, \omega_0) = \omega^*(r)$ for any ω_0, s_0, and r

Full Enforcement



Under **full enforcement**, the optimal allocation is **stationary** and the long-run distribution of ω is **degenerate** with mass at ω^*

Full Enforcement: Implementation through Taxes and Debt

- Consider one-period **state-contingent bond** $B_{r,t+1}$ in zero net supply and **taxes** $\mathcal{T}_t$ to repay **outstanding debt** D_t

$$D_t = \mathcal{T}_t + \sum_r q_{r,t+1} B_{r,t+1}$$

  where **state prices** are

$$q_{r,t+1} := \pi(r) M_{t,t+1} = \pi(r) \beta \frac{u'(e^o_{t+1} + B_{r,t+1})}{u'(e^y_t - \mathcal{T}_t - \sum_r q_{r,t+1} B_{r,t+1})}$$

- Let $d_t := \dfrac{D_t}{e_t s_t}$, $\tau_t := \dfrac{\mathcal{T}_t}{e_t s_t}$ and $b_{r,t+1} := \dfrac{B_{r,t+1}}{e_{t+1} s_{t+1}}$

Full Enforcement: Implementation through Taxes and Debt

- Fiscal reaction function:

$$\tau(d_t) = d_t - BR(d_t), \text{ with } BR(d_t) \equiv \frac{1}{s_t}\mathbb{E}_t\left[M_{t,t+1} s_{t+1} b_{r,t+1} \frac{e_{t+1}}{e_t}\right]$$

- In equilibrium, the SDF is:

$$M^*_{t,t+1} = \beta \frac{u'(e_{t+1} - C_{t+1})}{u'(C_t)} = \beta \frac{c^* e_t}{(1-c^*)e_{t+1}} = \delta \frac{e_t}{e_{t+1}}$$

- The yield curve is $y_t^k = -\frac{1}{k}\log(\mathbb{E}_t[M^*_{t,t+k}])$ under full risk sharing is:

$$y_t^k = -\log(\delta) + \log(\bar{\gamma}) > 0$$

Full Enforcement: Implementation through Taxes and Debt

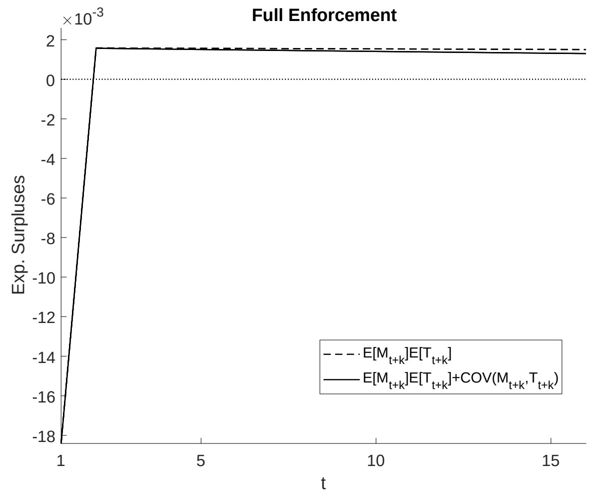
- The optimal state-contingent debt and taxes are stationary (no dependence on d_t) [check that it implements c^*!]:

$$b^*_{r,t+1} = 1 - \frac{1}{2}\frac{1}{s_{t+1}}$$
$$\tau^*_t = 1 - \frac{\delta}{s_t}\Big(\mathbb{E}_t[s_{t+1}] + \frac{1-\delta}{2\delta}\Big) \gtreqless 0$$

- The amount of debt sustained under the optimal policy [trasversality condition can be shown to be satisfied!]:

$$D^*_t = \sum_{j=0}^{\infty} \mathbb{E}_t[M^*_{t,t+k}\mathcal{T}^*_{t+k}]$$
$$D^*_t = \mathbb{E}_t[M^*_{t,t+j}] \cdot \mathbb{E}_t[\mathcal{T}^*_{t+j}] + \underbrace{COV_t[M^*_{t,t+j}, \mathcal{T}^*_{t+j}]}_{<0}$$

Full Enforcement: $\dfrac{D_1}{e_1} = 0.10$



Under full enforcement, a positive amount of debt can be sustained only if expected future surpluses

Optimal Constrained Efficient Allocation

Limited Enforcement

- The Planner value $V(s, \omega)$ is subject to both PK and PCy
- The first-order conditions are:

$$\frac{1 - \mathbf{c}(x)}{\mathbf{c}(x)} = \frac{1 + \lambda(x)}{1 + \mu(x)} \quad \text{and} \quad V_\omega(r, \mathbf{w}_r(x)) = -\mu(x)$$

The envelope condition is:

$$V_\omega(x) = -\lambda(x)$$

⇒ **Updating rule**:

$$V_\omega(r, \mathbf{w}_r(x)) = -\lambda(r, \mathbf{w}_r(x)) = -\mu(x)$$

Limited Enforcement

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$\Rightarrow$ **Updating rule**:

$$V_\omega(r, \mathbf{w}_r(x)) = -\lambda(r, \mathbf{w}_r(x)) = -\mu(x)$$

- There is $\omega^0(s) = \sup\{\omega | V_\omega(s, \omega) = 0\} \leq \omega^* \ \forall s$

$\Rightarrow$ If $\omega_0 \leq \omega^0(s_0)$ then $\lambda = 0$ and $c_0 = \mathbf{c}(s_0, \omega^0(s_0)) \geq \frac{1}{2}$

$\Rightarrow$ If $\omega_0 > \omega^0(s_0)$ then $\lambda > 0$ and $c_0 = \mathbf{c}(s_0, \omega_0) = 1 - \exp(\omega_0)$

Dynamics of Promised Utility

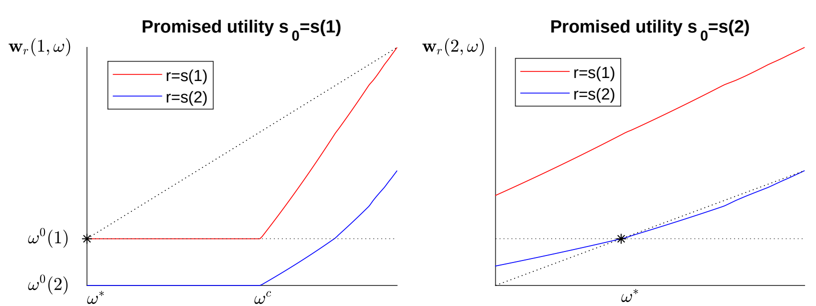
- The Planner would like to promise $\omega^0(s)$ to the current young
- But if PC_y is binding, the current young will refuse it
- In this case, the Planner must promise more to relax PC_y of the current young
- A higher promised utility means that more consumption must be delivered to next-period old, which tightens PC_y of future young

Dynamics of Promised Utility

Assume that c^* violates PC_y in at least one s and $s(1) \leq c^*$, the optimal policy $\mathbf{w}_r(s, \omega)$ is

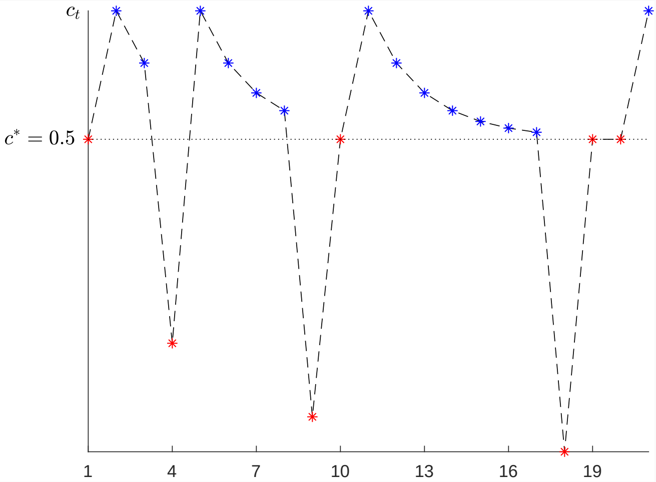
- increasing in ω
- increasing in s
- decreasing in r
- there is a critical $\omega^c > \omega^0(1)$ such that $\mathbf{w}_r(1, \omega) = \omega^0(r)$ if $\omega \leq \omega^c$ (**resetting**)
- there is a unique fixed point $\omega^f(s) = \mathbf{w}_s(s, \omega^f(s)) = \omega^* \; \forall s$

Dynamics of Promised Utility



Under **limited enforcement**, the optimal allocation is **history dependent** and the long-run distribution of ω is non degenerate in an ergodic set with countable infinite states

Dynamics of Consumption



Consumption of adjacent generations is serially **correlated**

Stochastic Discount Factor

- In an equilibrium model, the SDF is

$$M_{t,t+1} = \beta \frac{u'(e_{t+1} - C(\rho^{t+1}))}{u'(C(\rho^t))}$$

- In the sustainable optimal allocation, the SDF is

$$M_{t,t+1} = \underbrace{\delta \cdot \frac{\mathbf{c}(x_t)/(1+\mu(x_t))}{\mathbf{c}(x_{t+1})/(1+\mu(x_{t+1}))}}_{m_{t,t+1}} \cdot \frac{e_t}{e_{t+1}}$$

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$$M_{t,t+1} = \beta \frac{u'(e_{t+1} - C(\rho^{t+1}))}{u'(C(\rho^t))}$$

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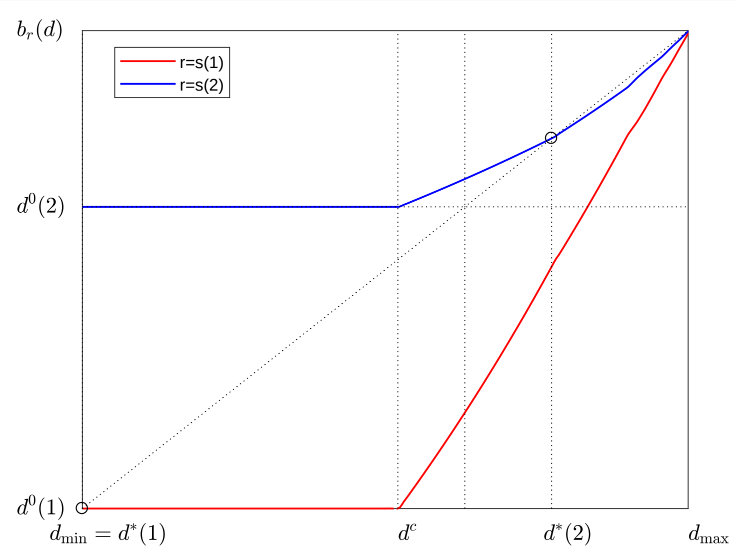
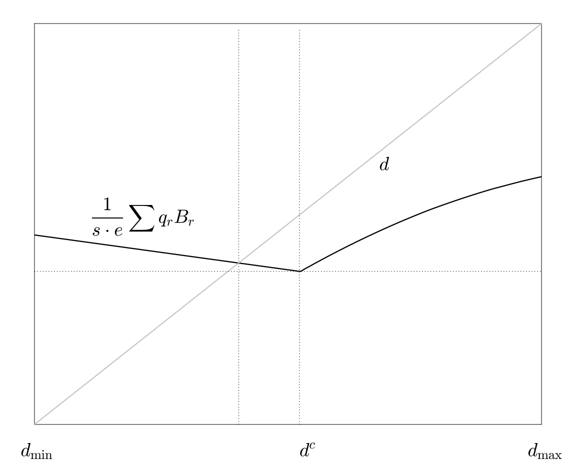
$$M_{t,t+1} = \underbrace{\delta \cdot \frac{\mathbf{c}(x_t)/(1+\mu(x_t))}{\mathbf{c}(x_{t+1})/(1+\mu(x_{t+1}))}}_{m_{t,t+1}} \cdot \frac{e_t}{e_{t+1}}$$

- Under **full enforcement**, the SDF is $M^*_{t,t+1} = \delta \frac{e_t}{e_{t+1}}$.
- Under **limited enforcement**, the SDF is decreasing in γ_{t+1}, increasing in s_{t+1} and decreasing (non-linearly) in ω_t

Dynamics of Debt

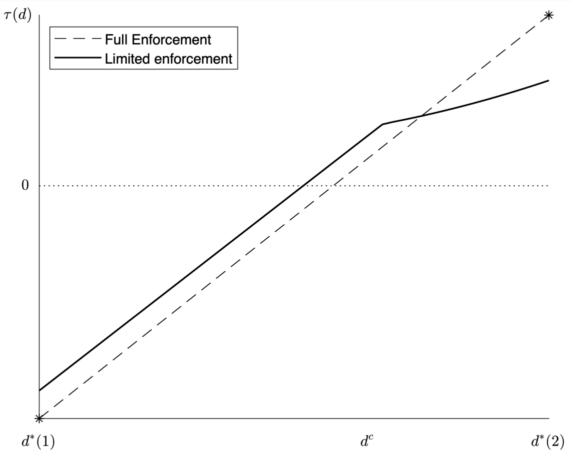


Bond Revenue



Below d^c bond revenue decreases since bond prices decrease and bond issuance is constant. Above d^c bond revenue can increase

Fiscal Reaction Function



The fiscal reaction function is non-linear in the outstanding debt:

a form of fiscal-fatigue

Debt Valuation

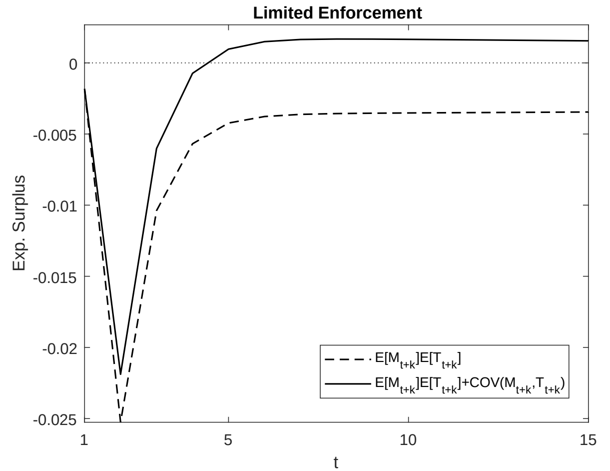
- Surplus $\mathcal{T}_{t+k}$ increases with both γ_{t+k} (pro-cyclical) and s_{t+k}
- The SDF $M_{t,t+k}$ decreases with γ_{t+k} (counter-cyclical) and increases with s_{t+k}

$\Rightarrow$ The $COV_t[M_{t,t+k}\mathcal{T}_{t+k}]$ can be decomposed in two terms

- ◇ A negative term related to variability of γ_t (risk-premium due to uninsured aggregate risk)
- ◇ A positive term related to variability of s_t (insurance-premium due to insured distributional risk)

- Overall $COV_t[M_{t,t+k}\mathcal{T}_{t+k}]$ can be positive if insurance premium sufficiently large

Limited Enforcement: $\dfrac{D_1}{e_1} = 0.10$



Under limited enforcement, a positive amount of debt can be sustained even if expected future deficits (debt as good hedge)

Main Take-Aways

- The observed high level of debt can be more sustainable than what (most of the) existing theories demonstrate
- The fiscal rules defining taxes, expenditure and bond issuance should however be designed optimally such that they respect limited enforcement constraints
- The optimal fiscal reaction function exhibits a form of fiscal fatigue
- Risk premium and insurance premium vary with debt and horizon: Can it be quantified using data?