Policies and Instruments for Self-Enforcing Agreements*

Bård Harstad† Francesco Lancia‡ Alessia Russo§

November 18, 2020

Abstract

International agreements must be self-enforcing. We characterize policies that support the best subgame-perfect equilibrium in a repeated game where firms invest in green technologies and consumers emit. Under a quantity agreement, emission is capped and countries must either overinvest in technology—to weaken the temptation to emit—or they must be punished unless they invest less—to maintain their willingness to retaliate on others. Under a price agreement, emission is taxed and investments subsidized. The price agreement dominates the quantity agreement because when firms are free to modify investment levels if another government defects, the punishment for defection is stronger.

Keywords: Climate change, repeated games, self-enforcing agreements, prices vs. quantities.

JEL: D86, F53, H87, Q54

*This research received funding from the European Research Council under the EU's 7th Framework Programme, ERC GA no. 283236.

†University of Oslo and the Frisch Centre. Email: bard.harstad@econ.uio.no

‡University of Salerno, CSEF, and CEPR. Email: francesco.lancia@unisa.it

§BI Norwegian Business School and CEPR. Email: alessia.russo@bi.no

The fundamental challenge in achieving international cooperation is to motivate countries to cooperate rather than free ride. In the absence of a world government, countries may need to rely on reputation and repeated play. However, it is doubtful whether the prospect of future cooperation is sufficient to motivate cooperation in the present. A country will be tempted to increase its emissions or to reduce its abatement effort, especially when immediate retaliation is unfeasible. This will be more tempting for countries that rely on fossil fuels, but less so for countries that have invested in renewable energy. Thus, the technology levels will influence the temptation to emit.

The importance of technology is recognized by practitioners. As stated by the IPCC (2014:1178): "*There is a distinct role for technology policy in climate change mitigation. This role is complementary to the role of policies aimed directly at reducing current GHG emissions.*"

This observation leads to a number of important questions. What are the characteristics of the best agreement, if the first best cannot be attained? Should countries be required to invest more in renewable energy, so as to be less tempted to emit or invest less, and therefore have the ability to credibly punish if others defect? Which policy implements the best self-enforcing agreement? Is a rigid quantity agreement better or worse than a flexible price agreement?

To answer these questions, Section I presents a repeated game in which private firms invest in green technology, such as renewables, in each period, before consumers choose fossil fuel consumption levels. Under the quantity agreement, analyzed in Section II, governments set emission quotas and investment mandates. Under the price agreement, analyzed in Section III, governments specify emission taxes and investment subsidies. To isolate the role of technology, we abstract from technological spillovers. As we justify below, we assume that countries would like to coordinate on the best subgame-perfect equilibrium (SPE) supported by a reversion to the "business-as-usual" Markov-perfect equilibrium (MPE).

Under a quantity agreement, there are two alternative types of distortions when the discount factor is too small to support the first best. Suppose, first, that technology is expensive relative to the cost of reducing consumption. In this case, the first-best investment levels are low and the greatest temptation is to deviate from the cooperative emission quantity. To motivate compliance when the discount factor is low, countries must be allowed to emit more than the first-best level. In order to mitigate the necessity to increase the emission level, it is optimal to require countries to invest more in renewables, that is, more than they would find to be optimal *conditional* on the equilibrium emission level. By requiring overinvestment in order that countries will have an overabundance of renewable energy, the temptation to defect by emitting more is weakened, and the necessary increase in the emission quantity (to motivate compliance) will be smaller.

In contrast, if technology is inexpensive, or if it is costly to reduce consumption, then the first-best investment level is high and the temptation to defect is greatest at the investment stage. To reduce this temptation, countries must be required to invest less than the levels they would find to be optimal given the agreed-upon emission quantity. In principle, for a fixed emission quantity, an increase in investment leads to Pareto improvements. This, however, limits the possibility to punish, i.e., by emitting more, if another country defects, thus making it tempting to defect at the investment stage. To reduce this temptation, countries must be punished so that they will invest less.

A price agreement produces a different set of incentives. When the discount factor is large, the first best is implemented by a Pigouvian tax and no investment subsidy. If a country defected at the investment policy stage, then it will have to discourage firms from investing. When this is observed by firms in other countries, they will realize that cooperation is about to end, and as a result emissions will increase and the demand for technology will fall. Thus, firms in non-deviating countries will also invest less, and in turn emission levels in those countries will be higher at the subsequent emission policy stage. For this reason, it is less tempting to defect at the investment policy stage under a price agreement than under a quantity agreement.

This logic also explains why it is always more tempting under a price agreement to defect at the emission policy stage than at the investment policy stage. The harder it is to motivate compliance (i.e., the smaller is the discount factor), the lower the equilibrium emission tax will need to be. To maintain the incentive to invest in this situation, green technology investment must be subsidized, thus reducing the temptation to emit and making it possible to motivate compliance without a large increase in the permitted emission quantity.

The comparison between the two designs uncovers a novel reason to justify why flexible instruments, such as the price agreement, can be preferred to the rigid quantity agreement. When technology is costly, the two designs are identical, but otherwise the price agreement is superior since the compliance constraints may bind at the investment stage under the quantity agreement, though not under the price agreement. Under a price agreement, firms in non-deviating countries invest less, and the subsequent punishment (i.e., the emission level) will be larger if a country defects. Under investment mandates, however, these firms will have to stick to that mandate.

Literature. The basic setup of the model follows Battaglini and Harstad (2016), which in turn draws on Harstad (2012; 2016). However, unlike the current model, they study MPEs and permit commitments to future contribution levels. Inspired by the literature on repeated games (see, Mailath and Samuelson, 2006), Barrett (1994; 2006), Dutta and Radner (2004; 2006), and Kerr, Lippert, and Lou (2020), among others, examine self-enforcing environmental agreements. However, they abstract from how technology can motivate compliance. This interaction has only been discussed previously by Harstad, Lancia, and Russo (2019) who restricted their attention to binary emission levels.[1] By allowing the emission levels to be continuous, the current analysis makes two important contributions.

First, we show that the best SPE does not necessarily require countries to overinvest in green technology (as in Harstad et al., 2019). When the technology is relatively inexpensive, one must instead punish countries for not reducing their investment in green technology. This links the study of self-enforcing environmental agreements to the industrial organization literature which looks at how firms can sustain collusion by overinvesting in capacity (see, Brock and Scheinkman, 1985, Benoit and Krishna, 1987, Davidson and Deneckere, 1990, and Compte, Frederic, and Rey, 2002). In that literature, firms overinvest in order to increase their ability to punish if another firm sells too much. This strategy is analogous to how countries may need to underinvest in green technology since that too enables the players to punish if someone defects. The strategies to underinvest (and thus increase the punishment) and to overinvest (and thus reduce one's own temptation) are mutually exclusive, but both are possible in our model, which unambiguously determines when one, rather than the other, is optimal.[2]

Second, the continuous emission level makes it meaningful to analyze the optimal price instrument. This analysis leads to the novel finding that the optimal investment subsidy is positive, even if there is no technological spillover, because technology weakens the temptation to emit. This result adds to the literature on whether two instruments are necessary in order to mitigate climate change (see, Acemoglu et al., 2012, and Golosov et al., 2014). Because a defection allows firms to react under a price agreement (by investing less), but not under a quantity agreement, we are also able to derive a novel advantage of the flexible price instrument, relative to the rigid quantity instrument. Thus, the model contributes to the important literature on the optimality of price regulation versus quantity regulation (see, Weitzman, 1974, and the subsequent literature).[3]

[1]In the relational contracting literature, Ramey and Watson (1997) and Halac (2015) show that technology investments can relax the compliance constraint on individual contributions to a public good; however, they focus on how an up-front investment by one party affects the hold-up problem.

[2]Fudenberg and Tirole (1984) also separate two strategic forces: the incentive to invest in the capacity to compete, and the incentive to reduce the competitors' aggression; however, they focus on entry deterrence rather than collusion.

[3]Hoel and Karp (2002) and Karp and Traeger (2020) analyze prices versus quantities in a dynamic setting with technology innovation and stock of pollution, but ignore strategic interactions between countries. Endres and Finus (2002) and Mideksa and Weitzman (2019) extend the prices versus quantities framework to a strategic setting, but neglet repeated interactions. Eichner and Pethig (2015) and Kornek and Marschinski (2018) compare prices versus quantities for environmental agreements, but consider a static coalition game and ignore the effects on compliance.

I. The Emission and Investment Game

A. *Countries and Payoffs*

Each country $i \in N \equiv \{1, ..., n\}$ is run by a benevolent government and has price-taking firms that invest in green technology—say, renewable energy—before they sell the energy they produce to consumers. At time $t \in \{1, 2, ...\}$, consumers in country i consume $y_{i,t}$ units of energy, where $r_{i,t}$ is from renewables and $g_{i,t}$ is from fossil fuels, such that $y_{i,t} = g_{i,t} + r_{i,t}$.

Variable $r_{i,t}$ can alternatively be interpreted as abatement technology, in which case $g_{i,t} = y_{i,t} - r_{i,t}$ is the actual emission level when $r_{i,t}$ units are abated. To simplify the analysis, we assume that $g_{i,t}$ and $r_{i,t}$ are perfect substitutes, there is no "brown" technology, and information is complete, assumptions that are relaxed in the Online Appendix.[4]

The benefit from country i's energy consumption is concave and increasing in y_i up to a bliss point, $\overline{y}$:

$$B(y_{i,t}) = -\frac{b}{2}(\overline{y} - y_{i,t})^2,$$

where $b > 0$ reflects the cost of reducing energy consumption. The bliss point represents the ideal energy level if emitting is costless. Thus, a country would never produce more than $\overline{y}$ due to the implicit costs of generating or transporting the energy. It is straightforward to allow for heterogeneity in $\overline{y}$.

While the actual emission is privately beneficial, the environmental cost to each country is $c \sum_{i \in N} g_{i,t}$, where $c > 0$ is the (present-discounted) marginal cost of emission. The cost of investing in technology is:

$$K(r_{i,t}) = \frac{k}{2} r_{i,t}^2,$$

where $k > 0$ is an investment cost parameter.

A government's objective is to maximize the present discounted value of its utility stream:

$$\sum_t \delta^t u(\mathbf{g}_t, r_{i,t}), \quad \text{with} \quad u(\mathbf{g}_t, r_{i,t}) \equiv B(g_{i,t} + r_{i,t}) - c \sum_{j \in N} g_{j,t} - K(r_{i,t}), \tag{1}$$

where $\delta \in (0, 1)$ is the common discount factor and $\mathbf{g}_t \equiv \{g_{1,t}, ..., g_{n,t}\} \in \mathbb{R}^n$ is the vector of emission levels.

[4] To simplify further, we do not require $g_{i,t}$ or $r_{i,t}$ to be positive. A negative $g_{i,t}$ may be feasible with carbon capture, for example. See, Harstad (2012) for how non-negative constraints can be taken into account.

Stocks. One can easily permit emissions and technology to be stocks that accumulate over time, rather than outputs that depreciate from one period to the next. If the stock of greenhouse gases is accumulating, c can represent the present-discounted cost of an additional (long-lasting) unit of emission. The technology $r_{i,t}$ can also be a stock since the analysis is unchanged if the marginal cost of adding to the stock is (proportionally) larger when the existing stock is large. For details, see Battaglini and Harstad (2016) who consider the utility function (1) even though they focus on coalition formation and MPEs.

B. *Quantities and Prices*

In each period, private firms choose how much to invest in renewables before consumers choose the energy consumption level. The sequential timing follows naturally when technology investment requires time to mature and become operational.

A government has the authority to regulate domestic emissions and technology investment using either *quantity* mandates or *price* instruments. In the case of the former, it sets emission and investment levels. In this case, the government is directly determining the variables of interest, and consumers and firms are not active players in the game. In the case of the latter, it specifies an emission tax, $\tau_{i,t} \in \mathbb{R}$, and an investment subsidy, $\varsigma_{i,t} \in \mathbb{R}$.

We assume that governments cannot commit to future policies. The investment subsidy is set just before the private sector's investment in green technology, while the emission tax is set just before the consumption of fossil fuel. The policies are implemented by all governments simultaneously and are observed by all. Hence, a crucial difference between the two types of regulations is that price instruments allow the private sector to flexibly adjust investment to the anticipation of future policies, while quantity mandates do not. (The role of timing is further discussed in Section IV.)

Taxes collected and subsidies paid by a government do not represent actual costs or revenues from the government's perspective. Their only effect is on the decisions to determine $g_{i,t}$ and $r_{i,t}$. After technology investments are made and carbon taxes are announced for period t, consumers choose the fossil fuel consumption level, $g_{i,t}$, so that the marginal benefit equals the emission tax:

$$B'\left(y_{i,t}\right) = \tau_{i,t} \Leftrightarrow g\left(\tau_{i,t}, r_{i,t}\right) = \overline{y} - r_{i,t} - \frac{\tau_{i,t}}{b}. \tag{2}$$

Naturally, the emission level is low when the tax is high or the country is endowed with green technology.

When investors can sell renewable energy to consumers, the marginal cost of investment in equilibrium equals the expected marginal benefit from consuming energy, plus the subsidy:

$$K'(r_{i,t}) = \mathbb{E}[B'(y_{i,t})] + \varsigma_{i,t} \Leftrightarrow r\left(\tau_{i,t}^{e}, \varsigma_{i,t}\right) = \frac{\tau_{i,t}^{e} + \varsigma_{i,t}}{k}, \tag{3}$$

where $\tau_{i,t}^{e}$ denotes the emission tax that is expected by the firms when they invest. A higher expected tax and a higher investment subsidy increase the firms' investments.

C. *Benchmarks and Equilibria*

Business-as-Usual. Under business-as-usual (BAU), country i's investment and emission policies are set non-cooperatively to maximize $u(\mathbf{g}_t, r_{i,t})$, as defined in (1). This outcome is equivalent to the unique MPE outcome of the repeated game and the unique SPE outcome of the stage game. At the emission stage, $u(\mathbf{g}_t, r_{i,t})$ is maximized when i emits:

$$g_b(r_{i,t}) \equiv \overline{y} - \frac{c}{b} - r_{i,t}. \tag{4}$$

The larger $r_{i,t}$ is, the smaller will be $g_b(r_{i,t})$, since the marginal benefit from emitting is small when i consumes renewable energy.

At the investment stage, $u(\mathbf{g}_t, r_{i,t})$ is maximized by the investment mandate:

$$r_b(g_{i,t}) \equiv \frac{b}{b+k}(\overline{y} - g_{i,t}).$$

Hence, the BAU investment and emission levels become:

$$r^b \equiv r_b\left(g^b\right) = \frac{c}{k} \quad \text{and} \quad g^b \equiv g_b\left(r^b\right) = \overline{y} - \frac{b+k}{bk}c. \tag{5}$$

According to (2) and (3), the government can also implement (5) by means of an emission tax and no investment subsidy:

$$\tau^b = c \quad \text{and} \quad \varsigma^b = 0,$$

since the firms anticipate $\tau_{i,t}^{e} = \tau^b$ when they invest. The utility for each country is $u^b \equiv u\left(\mathbf{g}^b, r^b\right)$, where $\mathbf{g}^b \equiv \left\{g^b, ..., g^b\right\} \in \mathbb{R}^n$.

First Best. In the first-best case, each government maximizes the sum of utilities. The first-best emission quota is:

$$g_*(r_{i,t}) \equiv \overline{y} - n\frac{c}{b} - r_{i,t},$$

and the first-best investment mandate is:

$$r_*(g_{i,t}) = r_b(g_{i,t}) = \frac{b}{b+k}(\overline{y} - g_{i,t}).$$

Hence, the first-best investment and emission levels become:

$$r^* \equiv r_* (g^*) = \frac{cn}{k} \quad \text{and} \quad g^* \equiv g_* (r^*) = \overline{y} - \frac{b+k}{bk} cn. \tag{6}$$

As in BAU, there is no need to regulate investment. Firms invest efficiently, conditional on the emission levels. Therefore, when using the price instrument, (2) and (3) imply that the government can implement (6) with a Pigouvian emission tax and no subsidy:

$$\tau^* = cn \quad \text{and} \quad \varsigma^* = 0,$$

since the emission tax anticipated by the firms is $\tau^e_{i,t} = \tau^*$. Clearly, $g^b > g^*$ and $r^b < r^*$ when $n > 1$. Because governments internalize the global emission cost in the first best, but not in BAU, the first-best utility is $u^* \equiv u(\mathbf{g}^*, r^*) > u^b$, where $\mathbf{g}^* \equiv \{g^*, ..., g^*\} \in \mathbb{R}^n$.

PROPOSITION 0: *In BAU, as well as in the first best, quantity mandates and price instruments are welfare-equivalent and it is unnecessary to regulate investment.*

Equilibria. The stage game has the structure of a multilateral prisoner's dilemma since all countries are better-off in the first best than in BAU. The stage game is slightly more complex than in a standard prisoner's dilemma because each period is described by an extensive-form stage game with sequential decisions. Nevertheless, a folk theorem holds and there are many SPEs when the discount factor is large. Because countries can communicate and coordinate and they are symmetric, we henceforth characterize the symmetric SPE that maximizes (1), under the assumption that any deviation from the equilibrium path triggers a permanent reversion to BAU. This assumption is made both because it enables us to illustrate the findings simply and pedagogically, and because this punishment may be realistic when the countries can observe a deviation but not necessarily the identity of the deviator.[5]

II. Quantity Agreements

For an agreement to be self-enforcing, and supported by an SPE, it must be preferable to comply rather than defect. That is, the discounted payoff given by (1), when all countries play their equilibrium strategies, must be larger than the payoff from free riding one period before reverting to BAU. Given the extensive-form stage game, this requirement implies that the equilibrium must satisfy one "compliance constraint" at the investment stage and another at the emission stage.

[5] Mailath and Samuelson (2006) explain not only why punishments can be stronger with min-max strategies, but also why they might be weaker if the SPE must be renegotiation proof. A careful exploration of the set of SPEs under alternative assumptions is beyond the scope of the present paper.

Compliance Constraints. Consider an equilibrium candidate in which every country emits g and invests r in every period. Each country's intertemporal value is a function of $(\mathbf{g}, r)$, with $\mathbf{g} \equiv \{g, ..., g\} \in \mathbb{R}^n$:

$$V(\mathbf{g}, r; \delta) = \frac{u(\mathbf{g}, r)}{1-\delta}. \tag{7}$$

First, consider the temptation to defect at the investment stage. It is easy to verify that the most attractive defection is r^b, given by (5). Let $V^r(\mathbf{r}_{-i}; \delta)$ represent a country's intertemporal value when its government deviates by choosing r^b while the governments of other countries comply with the equilibrium $\mathbf{r}_{-i} \equiv \{r, ..., r\} \in \mathbb{R}^{n-1}$. At the subsequent emission stage, all countries return to BAU. Thus, the defector will emit g^b, also given by (5), at the emission stage. This strategy leads to the continuation value $u^b/(1-\delta)$ for the deviator, plus the one-period gain thanks to the lower emission levels in the non-deviating countries induced by the fact that they have invested $r \neq r^b$. As in (4), $g_b(r)$ is the non-cooperative emission level once the equilibrium investment r is sunk. Thus,

$$V^r(\mathbf{r}_{-i}; \delta) = c(n-1)\left(g^b - g_b(r)\right) + \frac{u^b}{1-\delta}.$$

The compliance constraint at the investment stage is:

$$V(\mathbf{g}, r; \delta) \geq V^r(\mathbf{r}_{-i}; \delta). \tag{8}$$

Next, consider the temptation to free ride and defect at the emission stage. The payoff from investing r and then emitting $g_b(r)$ is:

$$V^g(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta) = -\frac{b}{2}(\overline{y} - [g_b(r) + r])^2 - c[g_b(r) + (n-1)g] - \frac{k}{2}r^2 + \frac{\delta u^b}{1-\delta},$$

where $\mathbf{g}_{-i} \equiv \{g, ..., g\} \in \mathbb{R}^{n-1}$. The compliance constraint at the emission stage is thus:

$$V(\mathbf{g}, r; \delta) \geq V^g(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta). \tag{9}$$

DEFINITION 1: *The optimal quantity agreement is the vector pair* $(\mathbf{g}, \mathbf{r})$ *that maximizes the objective* (7) *s.t. the compliance constraints* $(8)-(9)$.

If $\delta \to 1$, both (8) and (9) become $u(\mathbf{g}, r) \geq u^b$, which is trivially satisfied by any agreement that is better than BAU. Thus, when δ is sufficiently large, the first best can be sustained as the outcome of an SPE. When δ is smaller, long-term consequences become less important and it becomes tempting to defect at the investment or the emission stage. The following lemma establishes whether (8) or (9) binds first as δ falls. To formalize the condition, let $\delta_q^r(\mathbf{g}, \mathbf{r})$ and $\delta_q^g(\mathbf{g}, \mathbf{r})$ denote the levels of δ that solve $V(\mathbf{g}, r; \delta) =$

$V^r(\mathbf{r}_{-i};\delta)$ and $V(\mathbf{g},r;\delta) = V^g(\{g_b(r),\mathbf{g}_{-i}\},r;\delta)$, respectively, for any $(\mathbf{g},\mathbf{r})$, and define $\overline{\delta}_q^g \equiv \delta_q^g(\mathbf{g}^*,\mathbf{r}^*)$ and $\overline{\delta}_q^r \equiv \delta_q^r(\mathbf{g}^*,\mathbf{r}^*)$, where $\mathbf{r}^* \equiv \{r^*,...,r^*\} \in \mathbb{R}^n$.

LEMMA 1: *Since $\overline{\delta}_q^r = \frac{b-k}{2b}$ and $\overline{\delta}_q^g = \frac{k}{b+2k}$, then $\overline{\delta}_q^r \leq \overline{\delta}_q^g$ if and only if $\frac{k}{b} \geq \frac{1}{2}$.*

If $k/b \geq 1/2$, investments are costly and the cost-effective climate policy prioritizes a reduction in the consumption level. In this situation, it is more tempting to defect at the emission stage than at the investment stage, such that $\overline{\delta}_q^r \leq \overline{\delta}_q^g$. When $k/b < 1/2$, it is relatively costly to change the consumption level and the optimal investment levels are high. In this case, it is more tempting to defect at the investment stage than at the emission stage, such that $\overline{\delta}_q^r > \overline{\delta}_q^g$.

PROPOSITION 1: *The optimal quantity agreement is characterized by:*

i. *If $\delta \geq \max\left\{\overline{\delta}_q^g, \overline{\delta}_q^r\right\}$, all quantities are first best:*

$$g = g^* \quad \text{and} \quad r = r^*.$$

ii. *Suppose $k/b > 1/2$. If $\delta \in \left(\delta_q^r(\mathbf{g},\mathbf{r}), \overline{\delta}_q^g\right)$, the emission level is higher than in the first best, and the investment level is higher than is optimal, conditional on the emission level:*

$$g = g^* + \Lambda_q(\delta) > g_*(r) \quad \text{and} \quad r = r^* = r_*(g) + \frac{b}{b+k}\Lambda_q(\delta), \text{ where}$$
$$\Lambda_q(\delta) \equiv \frac{c(n-1)}{b}\left(1 - \delta - \sqrt{\frac{\delta(b+\delta k)}{k}}\right) > 0.$$

iii. *Suppose $k/b < 1/2$. If $\delta \in \left(\delta_q^g(\mathbf{g},\mathbf{r}), \overline{\delta}_q^r\right)$, the emission level is higher than in the first best, and the investment level is lower than is optimal, conditional on the emission level:*

$$g = g_*(r) = g^* + \Xi_q(\delta) \quad \text{and} \quad r = r^* - \Xi_q(\delta) < r_*(g) < r^*, \text{ where}$$
$$\Xi_q(\delta) \equiv \frac{c(n-1)}{k}\left(1 - \delta - \sqrt{\frac{k+\delta^2 b}{b}}\right) > 0.$$

iv. *If $\delta \leq \min\left\{\delta_q^g(\mathbf{g},\mathbf{r}), \delta_q^r(\mathbf{g},\mathbf{r})\right\}$, (8) and (9) both bind and together they determine $(\mathbf{g},\mathbf{r})$.*

A Folk Theorem. Part (i) confirms that the first-best outcome is achievable when δ is large.

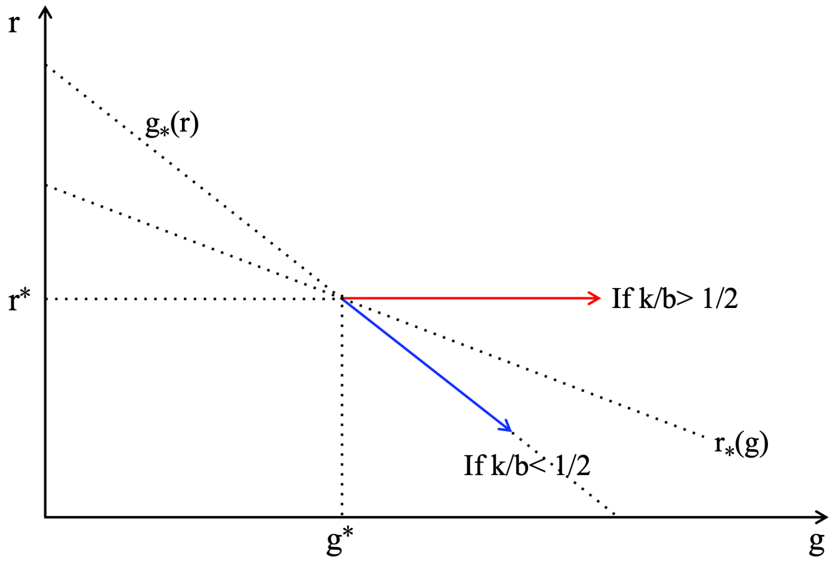

Figure 1: *As δ declines, the distortion follows the red arrow when $k/b > 1/2$ and the blue arrow when $k/b < 1/2$.*

Overinvestment in Green Technology. Part (ii) describes the optimal agreement when δ is too small to support the first best and $k/b > 1/2$. In this case, the compliance constraint at the emission stage binds and emission quotas must be larger than in the first best, leading to the distortion measured by $\Lambda_q(\delta) > 0$. With regard to the investment mandate, there exist two countervailing effects. On the one hand, a larger emission quantity discourages technology investment, since technology is a substitute for fossil fuel. On the other hand, by requiring firms to invest more in technology, the increase in the emission level necessary to motivate compliance is limited. The two effects cancel each other out, and $r = r^*$ remains unchanged as δ declines.[6] Compared to the conditional efficient level, i.e., $r_*(g)$, which is decreasing in g, it is evident that investments are distorted upward by the amount $(b/(b+k))\,\Lambda_q(\delta)$. If δ falls, the optimal quantity agreement changes, as illustrated by the horizontal (red) arrow in Figure 1.

Punishing Investment. When $k/b < 1/2$, as in part (iii), the declining (blue) arrow in Figure 1 illustrates how the optimal quantity agreement changes when δ falls. If b is large, or k is small, the cost-effective agreement prioritizes technology investment. The high level of investment makes it more tempting to deviate at the investment stage than at the emission stage. If δ is smaller, the compliance constraint requires that the investment be reduced and, in turn, emission levels will increase (even though g remains optimal conditional on r). This distortion is measured by $\Xi_q(\delta) > 0$. The inequality $r < r_*(g)$ implies that countries and firms must be punished if they invest as much as they find to be optimal given the level of g. This punishment is surprising at first glance, since for

[6]The two effects cancel each other out not because of the quadratic formulations, but because g and r are perfect substitutes in the utility function and the marginal cost of emission is constant.

a fixed level of g, it is a Pareto improvement to increase $r < r_* (g)$. Nevertheless, more investment cannot be allowed and must be punished by a reversion to BAU. Intuitively, if countries were permitted to invest the optimal amount, conditional on the agreed-upon g, then non-deviating countries would be unwilling to emit at a high level as a punishment following another country's deviation. Anticipating this unwillingness, countries would be tempted to defect at the investment stage.

COROLLARY 1: *When technology is inexpensive, each country invests less than it finds to be optimal given the emission level and is punished if it invests more.*

III. Price Agreements

We now consider the optimal agreement when governments set emission taxes and investment subsidies.

Compliance Constraints. At the emission policy stage, the compliance constraint is equivalent to (9) because, for any given technology level, there is a one-to-one mapping between the emission tax and the emission quantity. However, the compliance constraint at the investment policy stage differs from (8):

$$V(\mathbf{g}, r; \delta) \geq V^r(\widetilde{\mathbf{r}}_{-i}; \delta), \tag{10}$$

where $\widetilde{\mathbf{r}}_{-i} \equiv \{\widetilde{r}, ..., \widetilde{r}\} \in \mathbb{R}^{n-1}$ and $\widetilde{r} \equiv (\tau^b + \varsigma)/k$ is the investment level by firms in a non-deviating country after the firms have witnessed that another country has deviated from the equilibrium investment subsidy. After such a deviation, the firms anticipate that emissions are about to increase and that consumers' willingness to pay for renewable energy will shift from τ to τ^b. Therefore, firms in non-deviating countries will change their investment level to $\widetilde{r}$, obtained from (3) when $\tau^e = \tau^b$ while the investment subsidy remains at the equilibrium level. Thus, the subsequent emission level in a non-deviating country is $g_b = \varsigma/k$, such that:

$$V^r(\boldsymbol{\varsigma}_{-i}; \delta) = \frac{c(n-1)}{k}\varsigma + \frac{u^b}{1-\delta}. \tag{11}$$

When $\tau^b < \tau$, then $\widetilde{r} < r$, and $g_b(\widetilde{r}) > g_b(r)$. Thus, if a country defects at the investment policy stage, the subsequent emission level in a non-deviating country is larger than it would have been under a quantity agreement. There is therefore less temptation to defect at the investment policy stage under a price agreement than under a quantity agreement.

DEFINITION 2: *The optimal price agreement is the vector pair* $(\boldsymbol{\tau}, \boldsymbol{\varsigma})$*, with* $\boldsymbol{\tau} \equiv \{\tau, ..., \tau\} \in \mathbb{R}^n$ *and* $\boldsymbol{\varsigma} \equiv \{\varsigma, ..., \varsigma\} \in \mathbb{R}^n$*, that maximizes the objective* (7) *s.t. the private sector's response functions* (2) − (3) *and compliance constraints* (9) − (10).

If $\delta \to 1$, both compliance constraints become $u(\mathbf{g}, r) \geq u^b$, which is trivially satisfied by any agreement that is better than BAU. If δ declines below some threshold, $\overline{\delta}_p^g$, it will eventually be tempting to defect at the emission policy stage, since τ^* is not optimal from a national perspective. The threshold $\overline{\delta}_p^g$ is obtained by solving $V(\boldsymbol{\tau}^*, \boldsymbol{\varsigma}^*; \delta) = V^g(\boldsymbol{\tau}^*, \boldsymbol{\varsigma}^*; \delta)$, giving $\overline{\delta}_p^g = \overline{\delta}_q^g$. In addition, let $\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma})$ denote the threshold level of δ that solves $V(\boldsymbol{\tau}, \boldsymbol{\varsigma}; \delta) = V^r(\boldsymbol{\varsigma}_{-\boldsymbol{i}}; \delta)$ for any $(\boldsymbol{\tau}, \boldsymbol{\varsigma})$. When $\varsigma = 0$, (7), (10), and (11) imply that the compliance constraint at the investment policy stage reduces to $u(\mathbf{g}, r) \geq u^b$. Since $\varsigma^* = 0$, we obtain Lemma 2.

LEMMA 2: *If* $\delta > \overline{\delta}_p^g$, $(\boldsymbol{\tau}, \boldsymbol{\varsigma}) = (\boldsymbol{\tau}^*, \boldsymbol{\varsigma}^*)$ *and* $(9) - (10)$ *do not bind. As* δ *falls below* $\overline{\delta}_p^g$, (9) *binds before* (10).

The result in Lemma 2 is quite intuitive. Starting from the first best, the investment subsidy is zero. If a country attempts to further discourage its firms from investing, then investment falls in all countries and the deviator ends up with the BAU utility level.

PROPOSITION 2: *The optimal price agreement is characterized by:*

i. *If* $\delta \geq \overline{\delta}_p^g$, *the emission tax is Pigouvian and investments are unregulated:*

$$\tau = \tau^* \quad \text{and} \quad \varsigma = \varsigma^*.$$

ii. *If* $\delta \in \left(\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma}), \overline{\delta}_p^g\right)$, *the emission tax is smaller than the Pigouvian level, and investments are subsidized:*

$$\tau = \tau^* - \Lambda_p(\delta) \quad \text{and} \quad \varsigma = \Lambda_p(\delta), \text{ where } \Lambda_p(\delta) = b\Lambda_q(\delta);$$

iii. *If* $\delta \leq \delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma})$, (8) *and* (9) *both bind and together they determine* $(\boldsymbol{\tau}, \boldsymbol{\varsigma})$.

A Folk Theorem. Part (i) confirms that if δ is large, it is possible to implement the first-best climate policy.

Technology Subsidies. Part (ii) shows that if δ is small enough, then (9) binds and governments are tempted to defect by lowering the domestic emission tax. To mitigate this temptation, the required emission tax must be reduced. To maintain the incentive to invest, it becomes necessary to subsidize investment. As before, a higher investment level reduces the temptation to defect at the emission policy stage. Consequently, the increase in the equilibrium emission level necessary to motivate compliance is smaller than if investments were unregulated.

Figure 2 illustrates the optimal price agreement as a function of δ. When the compliance constraint at the emission policy stage starts to bind, the optimal emission tax

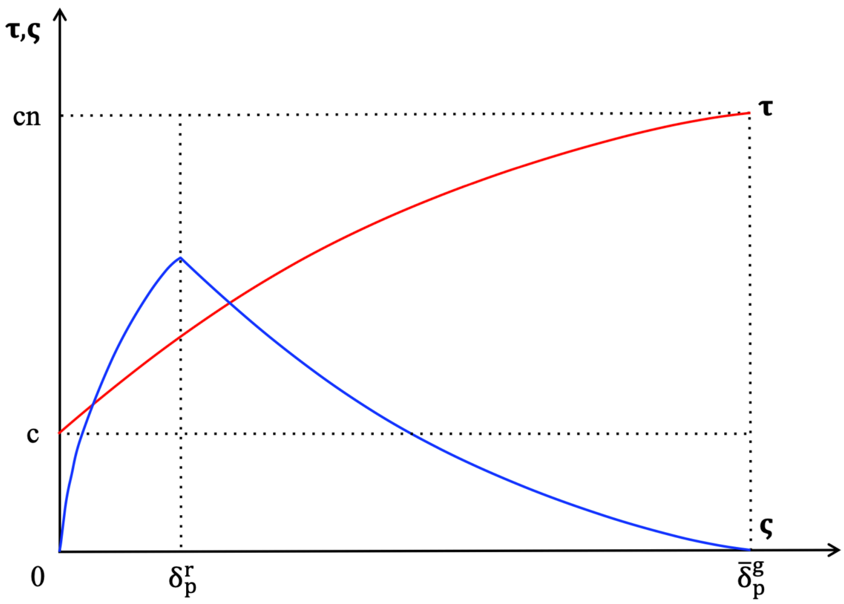

Figure 2: *The blue line represents the optimal investment subsidy while the red line represents the optimal emission tax, both as functions of the discount factor.*

is reduced below the Pigouvian level, but the optimal investment subsidy increases. If δ is even smaller, then part (iii) states that the compliance constraint at the investment policy stage may also bind. In this case, the optimal subsidy is reduced. In fact, $\varsigma \to 0$ when $\delta \to 0$, in the optimal price agreement.

Welfare Comparison. An important corollary is that payoffs under the price agreement are either equal to or higher than under the quantity agreement. When $k/b > 1/2$, the equilibrium g and r under the price agreement are identical to those under the quantity agreement. When $k/b < 1/2$, however, welfare is higher under the price agreement because it is less tempting to deviate from the optimal investment subsidy than from the investment mandate. Under a price agreement, firms in the non-deviating countries react to another country's deviation at the investment policy stage by reducing their own investment. Subsequently, emission levels will be higher, thus reducing the benefit from defecting. In contrast, investment choices under a quantity agreement are rigid and unresponsive to another country's recent deviation at the investment stage. Specifically, when $\delta \in \left[\overline{\delta}_p^g, \overline{\delta}_q^r\right]$, a price agreement can implement the first best, while a quantity agreement cannot. The welfare loss induced by quantities, relative to prices, increases as δ declines.

COROLLARY 2: *The price agreement Pareto dominates the quantity agreement because, under the former, firms react by investing less if a country defects, while under the latter, they cannot.*

IV. Further and Future Research

International agreements must be self-enforcing, but they can be designed in various ways. In our analysis, the optimal quantity agreement requires overinvestment in green technology if investments are expensive, but underinvestments if they are inexpensive. The optimal price agreement requires investment subsidies—even in the absence of technological spillovers—and the price agreement dominates the quantity agreement because it frees firms to react when another country defects.

The model is kept simple in order to make the analysis tractable and it can be extended in a number of directions. In the Online Appendix, we allow for imperfect monitoring, imperfect substitutability between fossil fuels and renewables, and "brown" technology.

In the presence of type I errors, countries might punish one another even when everyone has complied. In this case, it is desirable to limit the punishment phase, and if there is green technology investment, then the punishment phase can be shortened even further while maintaining the incentives to comply. When firms mistakenly react to what they believe are defections in other countries, we show that quantity agreements are superior when technology is expensive but, as shown above, price agreements are superior otherwise.

In the presence of type II errors, a defection might go unnoticed. This increases the temptation to defect also at the investment policy stage, and it might be necessary to tax investment in green technology. Thus, the result that cooperative investment might need to be punished generalizes to the situation of a price agreement when type II errors are possible.

The results hold and generalize also when we allow for environmentally damaging technology or if the green technology is an imperfect substitute for fossil fuel. The Online Appendix explains why the policy distortions vary non-linearly with the elasticity of substitution.

Other assumptions of the model are rather critical. If emission and investment policies are made simultaneously, then the two agreements are welfare-equivalent and the payoffs are lower than when the decisions are sequential. This is not surprising, since it is naturally more tempting to defect if one can do it twice, rather than only once, before countries retaliate.

The analysis offers a preliminary characterization of the best self-enforcing agreement. To keep the analysis tractable, we have restricted our attention to linear-quadratic utility functions, homogeneous countries, and stationary production functions, and have abstracted from fossil-fuel extraction and trade, adaptation to climate change, renegotiation, and smarter punishments than simply a reversion to the business-as-usual equilibrium. Future research should relax these assumptions in order to obtain a deeper understanding of optimal agreement design. Our primary contribution is a workhorse model sufficiently simple and tractable to be extended in these directions.

MATHEMATICAL APPENDIX

PROOF OF PROPOSITION 1:

Part (i) follows from Lemma 1. Part (ii) holds when $k/b > 1/2$ because in this case, $\overline{\delta}_q^g > \overline{\delta}_q^r$. For smaller δ that are close to $\overline{\delta}_q^g$, the optimization problem becomes:

$$\max_{g,r} \frac{1}{1-\delta}\left(-\frac{b}{2}\left(\overline{y}-[g+r]\right)^2 - cng - \frac{k}{2}r^2\right), \tag{12}$$

subject to (9) where $g_b(r) = \overline{y} - c/b - r$. Solving for r, we obtain $r = r^*$. Substituting the equilibrium r into (9) holding with equality and solving for g, yields:

$$g = g^* + \Lambda_q(\delta), \text{ where } \Lambda_q(\delta) \equiv \frac{c(n-1)}{b}\left(1-\delta-\sqrt{\frac{\delta(b+\delta k)}{k}}\right)$$

and $\partial\Lambda_q(\delta)/\partial\delta < 0$. Hence, $r = r^* = r_*(g) + (b/(b+k))\Lambda_q(\delta)$, where $r_*(g) = (b/(b+k))(\overline{y}-g)$. Part (iii) holds when $k/b < 1/2$ because in this case, $\overline{\delta}_q^g < \overline{\delta}_q^r$. For smaller δ that are close enough to $\overline{\delta}_q^r$, the optimization problem consists in maximizing (12), subject to (8) where $g^b = \overline{y} - c/b - r^b$. Solving for r, we obtain $r = \overline{y} - cn/b - g$. Substituting the equilibrium r into (8) holding with equality and solving for g, yields:

$$g = g^* + \Xi_q(\delta), \text{ where } \Xi_q(\delta) = \frac{c(n-1)}{k}\left(1-\delta-\sqrt{\frac{k+\delta^2 b}{b}}\right)$$

and $\partial\Xi_q(\delta)/\partial\delta < 0$. Hence, $r = r^* - \Xi_q(\delta)$. Part (iv) requires (8) and (9) to be simultaneously satisfied when $\delta \leq \min\left\{\delta_q^g(\mathbf{g},\mathbf{r}), \delta_q^r(\mathbf{g},\mathbf{r})\right\}$, where $\delta_q^r(\mathbf{g},\mathbf{r})$ is obtained by solving (8) holding with equality with respect to δ for $(\mathbf{g},\mathbf{r})$ determined in part (ii) and $\delta_q^g(\mathbf{g},\mathbf{r})$ is obtained by solving (9) holding with equality with respect to δ for $(\mathbf{g},\mathbf{r})$ determined in part (iii). As $\delta \to 0$, the optimal quantities approach $g = g^b$ and $r = r^b$.

PROOF OF PROPOSITION 2:

Part (i) follows from Lemma 2. To demonstrate part (ii), we solve the following optimization problem, for smaller δ that are close to $\overline{\delta}_p^g$:

$$\max_{\tau,\varsigma} \frac{1}{1-\delta}\left(-\frac{b}{2}\left(\overline{y}-[g(\tau,\varsigma)+r(\tau,\varsigma)]\right)^2 - cng(\tau,\varsigma) - \frac{k}{2}\left(r(\tau,\varsigma)\right)^2\right),$$

subject to $g(\tau,\varsigma) = \overline{y} - (b\varsigma + (b+k)\tau)/bk$, $r(\tau,\varsigma) = (\varsigma+\tau)/k$ and (9) where $g_b(\tau,\varsigma) = \overline{y} - c/b - r(\tau,\varsigma)$. Solving for ς, we obtain $\varsigma = \tau^* - \tau$. Substituting the equilibrium ς into

(9) holding with equality and solving for τ, yields:

$$\tau = \tau^* - \Lambda_p(\delta), \text{ where } \Lambda_p(\delta) = c(n-1)\left(1 - \delta - \sqrt{\frac{\delta(b+\delta k)}{k}}\right)$$

and $\partial\Lambda_p(\delta)/\partial\delta < 0$. Hence, $\varsigma = \Lambda_p(\delta)$. Part (iii) requires (9) and (11) to be simultaneously satisfied when $\delta < \delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma})$, where $\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma})$ is obtained by solving (11) holding with equality with respect to δ for $(\boldsymbol{\tau}, \boldsymbol{\varsigma})$ determined in part (ii). As $\delta \to 0$, the optimal prices approach $\tau = \tau^b$ and $\varsigma = \varsigma^b$.

REFERENCES

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous. 2012. "The Environment and Directed Technical Change." *American Economic Review*, 102, 131–166.

Barrett, Scott. 1994. "Self-Enforcing International Environmental Agreements." *Oxford Economic Papers*, 46, 878–894.

Barrett, Scott. 2006. "Climate Treaties and 'Breakthrough' Technologies." *American Economic Review, Papers and Proceedings*, 96, 22–25.

Battaglini, Marco and Bård Harstad. 2016. "Participation and Duration of Environmental Agreements." *Journal of Political Economy*, 124, 160–204.

Benoit, Jean-Pierre and Vijay Krishna. 1987. "Dynamic Duopoly: Prices and Quantities." *Review of Economic Studies*, 54, 23–36.

Brock, William A. and José A. Scheinkman. 1985. "Price Setting Supergames with Capacity Constraints." *Review of Economic Studies,* 52, 371–382.

Compte, Olivier, Jenny Frédéric, and Patrick Rey. 2002. "Capacity Constraints, Mergers and Collusion." *European Economic Review*, 46, 1–29.

Davidson, Carl and Raymond Deneckere (1990). "Excess Capacity and Collusion." *International Economic Review*, 31, 521–541.

Dutta, Prajit K. and Roy Radner. 2004. "Self-Enforcing Climate-Change Treaties." *Proceedings of the National Academy of Science of the United States of America*, 101, 5174–5179.

Dutta, Prajit K. and Roy Radner. 2006. "Population Growth and Technological Change in a Global Warming Model." *Economic Theory*, 29, 251–270.

Eichner, Thomas, and Rudiger Pethig. 2015. "Self-enforcing International Environmental Agreements and Trade: Taxes versus Caps." *Oxford Economic Papers*, 67, 897–917.

Endres, Alfred and Michael Finus. 2002. "Quotas may Beat Taxes in a Global Emission Game." *International Tax and Public Finance*, 9, 687–707.

Fudenberg, Drew and Jean Tirole. 1984. "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look." *American Economic Review*, 74, 361–366.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski. 2014. "Optimal Taxes on Fossil Fuel in General Equilibrium." *Econometrica*, 82, 41–88.

Halac, Marina. 2015. "Investing in a Relationship." *RAND Journal of Economics*, 46, 165–185.

Harstad, Bård. 2012. "Climate Contracts: A Game of Emissions, Investments, Negotiations, and Renegotiations." *Review of Economic Studies*, 79, 1527–1557.

Harstad, Bård. 2016. "The Dynamics of Climate Agreements." *Journal of the European Economic Association*, 14, 719–752.

Harstad, Bård, Francesco Lancia, and Alessia Russo. 2019. "Compliance Technology and Self-enforcing Agreements." *Journal of the European Economic Association*, 19, 1–29.

IPCC. 2014. Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, Cambridge University Press.

Hoel, Michael and Larry Karp. 2002. "Taxes versus Quotas for a Stock Pollutant." *Resource and Energy Economics*, 24, 367–384.

Karp, Larry S. and Christian P. Traeger. 2020. "Prices Versus Quantities Reassessed." *Econometric Modeling: Agriculture*, forthcoming.

Kerr, Suzi, Steffen Lippert, and Edmund Lou. 2020. "Financial Transfers and Climate Cooperation." Mimeo.

Kornek, Ulrike and Robert Marschinski. 2018. "Prices vs. Quantities for International Environmental Agreements." *Oxford Economic Papers*, 70, 1084–1107.

Mailath, George J. and Larry Samuelson. 2006. "Repeated Games and Reputations." Oxford University Press.

Mideksa, Torben K. and Martin L. Weitzman. 2019. "Prices versus Quantities across Jurisdictions." *Journal of the Association of Environmental and Resource Economists*, 6, 883–891.

Ramey, Garey and Joel Watson. 1997. "Contractual Fragility, Job Destruction, and Business Cycles." *Quarterly Journal of Economics*, 112, 873–911.

Weitzman, Martin L. 1974. "Prices vs. Quantities." *Review of Economic Studies,* 41, 477–491.

ONLINE APPENDIX

These appendices present supplementary material referenced in the paper. Appendix A introduces the possibility of investing in brown technologies in addition to green technologies. Appendix B characterizes the optimal self-enforcing quantity and price agreements under imperfect monitoring.

APPENDIX A. MULTIPLE TECHNOLOGIES

In the baseline model, country can invest only in green technology, which reduces the marginal value of emitting. In reality, energy can be produced from various sources, some of which are brown, such as drilling technology, which is beneficial in the extraction and consumption of fossil fuel and therefore complementary to emitting.[1] We can then expand the baseline model by permitting countries to invest in a technology portfolio, which includes clean and renewable technology r_{Ci} and brown technology r_{Bi}. The benefit function of country i is modified as follows:

$$B\left(g_i, r_{Bi}, r_{Ci}\right) = -\frac{b}{2}\left(\overline{y} - \left[g_i + r_{Ci}\right]\right)^2 - \frac{q}{2}\left(g_i - r_{Bi}\right)^2 + z r_{Ci} r_{Bi},$$

with $q > 0$ and $z \in \left(\underline{z}, \overline{z}\right)$ measures the elasticity of substitution between the two types of technologies. The term $-\left(q/2\right)\left(g_i - r_{Bi}\right)^2$ represents the cost of extracting fossil fuel beyond the capacity level r_{Bi}. The capacity to provide fossil fuel is brown since it reduces the cost of providing g_i and thus increases the net benefit from consuming fossil fuel. Brown technologies and clean technologies can be interdependent. The parameter z is positive when energy produced from, for example, solar or wind power is complemented by traditional fossil fuel sources in order to ensure a constant flow of electricity. In contrast, the case of substitute technologies arises when $z \leq 0$. We allow the cost of investment $\left(k_\sigma/2\right) r_{\sigma i}^2$ for each $\sigma \in \{C, B\}$, where $k_\sigma > 0$, to vary across technologies.

LEMMA A.1: *When $n > 1$, we have $g^* < g^b$, $r_B^* > (\leq)\, r_B^b$ for $z > (\leq)\, (b + k_C)q/b$ and $r_C^* > (\leq)\, r_C^b$ for $z < (\geq)\, (q + k_B)b/q$.*

Clearly, the first-best level of emissions is lower than the BAU level, since the global emission cost is internalized only in the first best. The first-best levels of investment in technologies, however, can be higher or lower than the BAU levels depending on the type of technology and on the degree of interdependence between types. If technologies are

[1] In Harstad, Lancia, and Russo (2019), we studied how technologies of different types affect incentives to comply with emissions when g is a binary variable, and thus we abstracted from climate policy implications.

substitutes, namely $z \leq 0$, then there will be more investment in clean technologies and less in brown ones in the first-best case relative to the case in which neither emissions nor investment are internationally negotiated. If technologies are complements, namely $z > 0$, then investment in both types of technologies must be either larger (when $z > (b+k_C)q/b$) or smaller (when $z \geq (q+k_B)b/q$) in the first best than in BAU.[2]

We have shown that global welfare is higher in a price agreement than in a quantity agreement in the absence of uncertainty. Clearly, the result also holds true in the case of multiple technologies. Therefore, we focus here on the characterization of the optimal self-enforcing agreements when national governments use price instruments. As shown in the paper, it is optimal to implement a Pigouvian emission tax equal to $\tau^* = cn$ and investment subsidies for brown and clean technologies equal to $\varsigma_B^* = \varsigma_C^* = 0$, when δ is sufficiently large. As δ becomes smaller, emissions must be allowed to increase in order to provide incentives for governments to comply with the agreement by reducing the emission tax. However, the dampening of the emission tax is partially mitigated by the introduction of subsidies for investment in clean technologies and by taxes imposed on firms investing in dirty technologies. The fiscal distortion generated by the friction of limited enforcement is quantified in the following proposition.

PROPOSITION A.1: *If* $\delta \in (\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma_B}, \boldsymbol{\varsigma_C}), \overline{\delta}_p^g]$, *the optimal price agreement is characterized by:*

$$\tau^* - \tau = \Lambda_m(\delta, z) > 0,$$

and

$$\varsigma_B - \varsigma_B^* = -(q/(b+q))\Lambda_m(\delta, z) \;\; \textit{and} \;\; \varsigma_C - \varsigma_C^* = (b/(b+q))\Lambda_m(\delta, z),$$

where $\partial \Lambda_m(\delta, z)/\partial \delta < 0$ *and*

$$\frac{\partial \Lambda_m(\delta, z)}{\partial z} \begin{cases} \geq 0 & \textit{if} \quad z \leq \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\} \\ < 0 & \textit{if} \quad z > \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\} \end{cases}.$$

Proposition A.1 provides new insights by generalizing the results of Proposition 2 in the paper to an environment in which countries invest in a technology portfolio. Tax distortions from first-best policies, denoted by $\Lambda_m(\delta, z)$, are greater when the discount rate is lower. Such distortions are also affected by the elasticity of substitution between brown technologies and clean technologies in a non-linear way. In particular, $\Lambda_m(\delta, z)$ is the largest when $z = \min\{(b+k_C)q/b, (q+k_B)b/q\}$, since either $r_B^* = r_B^b$ or $r_C^* = r_C^b$. Intuitively, when $r_\sigma^* = r_\sigma^b$ for some σ, it is optimal to implement policies so that the domestic private sector responds by investing in technology σ at the first-best level,

[2] The former occurs when $q/b < \sqrt{(q+k_B)/(b+k_C)}$, that is, when the use of clean technologies is more effective than the use of brown technologies, while the latter occurs in the opposite case.

i.e., $r_\sigma(\tau, \varsigma_B, \varsigma_C) = r_\sigma^*$, which restricts the ability of the two complementary investment policies, ς_B and ς_C, to curb fiscal distortions.[3]

MATHEMATICAL APPENDIX A

PROOF OF LEMMA A.1:

The first-best levels of emission and investment in brown and clean technologies are obtained by solving:

$$\max_{g,r_B,r_C} -\frac{b}{2}\left(\overline{y} - [g + r_C]\right)^2 - \frac{q}{2}\left(g - r_B\right)^2 + zr_Cr_B - cng - \frac{k_B}{2}r_B^2 - \frac{k_C}{2}r_C^2,$$

and are equal to:

$$\begin{aligned}
g^* &= \left(\left((q + k_B)k_C + (q - z)\,z\right)\overline{y}b + cn\left(z^2 - (q + k_B)(b + k_C)\right)\right)/\varrho,\\
r_B^* &= \left((z + k_C)\,\overline{y}bq - cn\left((b + k_C)q - bz\right)\right)/\varrho, \text{ and}\\
r_C^* &= \left((z + k_B)\,\overline{y}bq + cn\left((q + k_B)b - qz\right)\right)/\varrho,
\end{aligned}$$

where $\varrho \equiv (b + q)\left(k_Bk_C - z^2\right) + bq\left(2z + k_B + k_C\right) > 0$ for $z \in (\underline{z}, \overline{z})$ with

$$\begin{aligned}
\underline{z} &\equiv \frac{bq - \sqrt{\left(bq + (b + q)\,k_B\right)\left(bq + (b + q)\,k_C\right)}}{b + q} \text{ and}\\
\overline{z} &\equiv \frac{bq + \sqrt{\left(bq + (b + q)\,k_B\right)\left(bq + (b + q)\,k_C\right)}}{b + q}.
\end{aligned}$$

Since $z < \overline{z}$, then $g^b > g^*$. Furthermore, $r_B^*(n) > (\leq)\, r_B^*(1) = r_B^b$ when $z > (\leq)\,(b + k_C)q/b$ and $r_C^*(n) > (\leq)\, r_C^*(1) = r_C^b$ when $z < (\geq)\,(q + k_B)b/q$.

PROOF OF PROPOSITION A.1:

The optimal self-enforcing price agreement is obtained by solving $\max_{\boldsymbol{\chi}} V(\boldsymbol{\chi}; \delta)$, where $\boldsymbol{\chi} \equiv (\boldsymbol{\tau}, \boldsymbol{\varsigma_B}, \boldsymbol{\varsigma_C})$ and $V(\boldsymbol{\chi}; \delta)$ is equal to:

$$-\frac{1}{1-\delta}\left(\frac{b}{2}\left(\overline{y} - [g + r_C(\chi)]\right)^2 + \frac{q}{2}\left(g - r_B(\chi)\right)^2 - zr_C(\chi)\,r_B(\chi) + cng(\chi) + \sum_\sigma \frac{k_\sigma}{2}r_\sigma^2(\chi)\right)$$

[3] Acemoglu et al. (2016) develop a growth model in which dirty and clean technologies compete in each of many product lines. They find that a shift toward clean technology is possible only when the two energy technologies are not complementary. In contrast, we find that global emissions can be reduced also when technologies are complementary by means of an optimal combination of taxes on dirty technology and subsidies for clean technology, whose respective amounts depend on the degree of complementarity between the two types of technologies.

subject to:

$$V(\boldsymbol{\chi};\delta) \geq V^g(\boldsymbol{\chi};\delta) = -\frac{b}{2}(\overline{y} - [g_b(\cdot) + r_C(\chi)])^2 - \frac{q}{2}(g_b(\cdot) - r_B(\chi))^2 + zr_B(\chi)r_C(\chi)$$
$$-c(n-1)g(\chi) - cg_b(\cdot) - \sum_\sigma \frac{k_\sigma}{2} r_\sigma^2(\chi) + \frac{\delta}{1-\delta}u^b,$$

where $g_b(\cdot) = (b\overline{y} - br_C + qr_B - c)/(q+b)$, and

$$V(\boldsymbol{\chi};\delta) \geq V^r(\boldsymbol{\chi}_{-i};\delta) = c(n-1)\left(g^b - g_b(\cdot)\right) + \frac{u^b}{1-\delta}, \text{ where} \tag{A.1}$$

$$u^b = -\frac{b}{2}\left(\overline{y} - \left[g^b + r_C^b\right]\right)^2 - \frac{q}{2}\left(g^b - r_B^b\right)^2 + zr_B^b r_C^b - cng^b - \sum_\sigma \frac{k_\sigma}{2}\left(r_\sigma^b\right)^2(\chi),$$

$g^b = g^*$, $r_B^b = r_B^*$ and $r_C^b = r_C^*$ for $n = 1$. The optimal private responses $g(\chi)$, $r_B(\chi)$ and $r_C(\chi)$ are obtained by solving:

$$b(\overline{y} - (g + r_C)) - q(g - r_B) - \tau = 0, \tag{A.2}$$
$$q(g - r_B) + zr_C - k_B r_B + \varsigma_B = 0, \tag{A.3}$$
$$b(\overline{y} - (g + r_C)) + zr_B - k_C r_C + \varsigma_C = 0. \tag{A.4}$$

The function $g_b(\cdot) = (b\overline{y} - b\widetilde{r}_C + q\widetilde{r}_B - c)/(q+b)$ in (A.1) is the optimal domestic level of emission conditioned on investment $\widetilde{r}_B$ and $\widetilde{r}_C$, which are obtained by solving (A.2)-(A.4) when $\tau = \tau^b$. Replacing g^b and $g_b(\cdot)$ into (A.1), we obtain:

$$V(\boldsymbol{\chi};\delta) \geq V^r(\boldsymbol{\chi}_{-i};\delta) = \frac{c(n-1)}{\varrho}\left(b((-q+z)\varsigma_B + (q+k_B)\varsigma_C) - q(k_C\varsigma_B + z\varsigma_C)\right) + \frac{u^b}{1-\delta}.$$

Since $\varsigma_B^* = \varsigma_C^* = 0$, we have that $V^r\left(\boldsymbol{\chi}_{-i}^*;\delta\right) = u^b/(1-\delta)$ with $\boldsymbol{\chi}^* \equiv (\boldsymbol{\tau}^*, \boldsymbol{\varsigma}_B^*, \boldsymbol{\varsigma}_C^*)$, which implies that $V(\boldsymbol{\chi}^*;\delta) > V^r\left(\boldsymbol{\chi}_{-i}^*;\delta\right)$ is always satisfied. Let $\delta_p^g(\boldsymbol{\chi})$ denote the level of δ that solves $V(\boldsymbol{\chi};\delta) = V^g(\boldsymbol{\chi};\delta)$ and $\overline{\delta}_p^g \equiv \delta_p^g(\boldsymbol{\chi}^*)$, which is equal to:

$$\overline{\delta}_p^g = \frac{(b+q)k_B k_C + bq(k_B + k_C) + 2bqz - (b+q)z^2}{b^2(q+k_B) + q((2k_B+q)k_C - 2z^2) + b((2k_C+q)q + 2((q+k_C)k_B - (z-q)z))}.$$

If $\delta > \overline{\delta}_p^g$, then neither the compliance constraint at the emission policy stage nor the compliance constraint at the investment policy stage binds when taxes and subsidies are set to χ^*. Hence, the optimal self-enforcing price agreement is characterized by $\tau = \tau^*$ and $\varsigma_B = \varsigma_C = 0$. If $\delta \leq \overline{\delta}_p^g$, then the compliance constraint at the emission policy stage binds when $\chi = \chi^*$. Since $\partial V^g(\boldsymbol{\chi};\delta)/\partial\varsigma_B = \partial V(\boldsymbol{\chi};\delta)/\partial\varsigma_B$ and $\partial V^g(\boldsymbol{\chi};\delta)/\partial\varsigma_C = \partial V(\boldsymbol{\chi};\delta)/\partial\varsigma_C$, the equilibrium policies $\varsigma_B(\tau)$ and $\varsigma_C(\tau)$ are obtained by solving $\partial V(\boldsymbol{\chi};\delta)/\partial\varsigma_B = 0$ and $\partial V(\boldsymbol{\chi};\delta)/\partial\varsigma_C = 0$ and are equal to $\varsigma_B(\tau) =$

$(q/(b+q))(\tau - cn)$ and $\varsigma_C(\tau) = (b/(b+q))(cn-\tau)$. Substituting $\varsigma_B(\tau)$ and $\varsigma_C(\tau)$ into $V(\boldsymbol{\chi};\delta) = V^g(\boldsymbol{\chi};\delta)$ and solving for τ, we obtain:

$$\begin{aligned}
\tau &= \tau^* - \Lambda_m(\delta,z)\,, \text{ where} \\
\Lambda_m(\delta,z) &= (n-1)\,c\left((1-\delta) + \sqrt{\frac{\delta q(k_C(q+k_B\delta)-\delta z^2)}{(b+q)\,k_Bk_C + qb\,(k_B+k_C) + 2bqz - (b+q)z^2}}\right) \\
&\quad - (n-1)\,c\sqrt{\frac{\delta\left((b^2(q+k_B)+bq(q-2z)) + b\delta(qk_C + k_B(q+k_C) - (z-2q)\,z)\right)}{(b+q)\,k_Bk_C + qb\,(k_B+k_C) + 2bqz - (b+q)z^2}},
\end{aligned}$$

$\Lambda_m(\delta,z) > 0$, $\partial\Lambda_m(\delta,z)/\partial\delta < 0$ and

$$\frac{\partial\Lambda_m(\delta,z)}{\partial z}\begin{cases} \geq 0 & if \quad z \leq \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\} \\ < 0 & if \quad z > \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\}\end{cases}.$$

The level $\delta^r(\boldsymbol{\tau},\boldsymbol{\varsigma_B},\boldsymbol{\varsigma_C})$ is obtained by solving $V(\boldsymbol{\chi};\delta) = V^r(\boldsymbol{\chi_{-i}};\delta)$ with respect to δ for equilibrium levels $(\boldsymbol{\tau},\boldsymbol{\varsigma_B},\boldsymbol{\varsigma_C})$.

APPENDIX B: TECHNOLOGY AND IMPERFECT TRANSPARENCY

The baseline model assumes perfect information and no uncertainty. In reality, domestic climate policies are not as transparent as other policies, such as for example trade tariffs. Hence, the uncertainty surrounding the implementation of climate policies is likely to be the cause of disagreements across countries. In this appendix, we therefore permit domestic climate policies to be imperfectly observed and show that: (i) motivating compliance may require punishing investments in renewables also under price regulation by means of investment taxes; and (ii) the welfare achieved by means of quantity controls may be higher than that achieved by means of price instruments.

To highlight the effect of uncertainty in the simplest way, we assume that a country's domestic policy actions can sometimes be misperceived by other countries. Accordingly, we introduce a shock at each stage, which occurs after climate policy decisions have been made, but before private sector decisions have been made. This uncertainty leads to two types of errors: a *type I error* occurring with probability λ, due to a country incorrectly perceiving that another has defected from the negotiated agreement, even if it has not; a *type II error* occurring with probability η, due to the fact that when a country is defecting, the defection goes undetected.[4]

[4] We assume that shocks occurring at the investment and emission policy stages are independently drawn from the same distribution and do not depend on the type of policy instruments employed. Furthermore, we assume that the private sector perfectly observes its own country's climate policy actions, while it misperceives those of other countries to the same extent as its country's government. This

For the sake of comparison, we focus on the optimal self-enforcing agreements in which deviations are punished by reverting to BAU. Given that the punishment can be triggered by mistake due to uncertainty, however, failures of climate treaties through punishment reversion can be temporary. To show how the presence of uncertainty modifies the optimal self-enforcing agreement, in what follows we separately analyze the case of type I error and the case of type II error. Clearly, the overall effect of uncertainty should be viewed as a combination of the effects of the two types of error.

B.I. Type II Errors

We start by considering the scenario in which $\lambda = 0$ and $\eta > 0$. In this case, undeserved punishments will *not* be triggered along the equilibrium path, since there cannot be any misperceived detection of deviations when countries comply.

Compliance Constraints. In the absence of a costly punishment, the optimal self-enforcing agreement can be achieved by imposing an infinite duration of punishment, that is, $T = \infty$, which maximizes retaliatory power without reducing welfare. Hence, the intertemporal value of cooperation is $V(\mathbf{g}, r; \delta) = u(\mathbf{g}, r) / (1 - \delta)$.

The compliance constraint at the emission policy stage is equivalent to the constraint reported in the paper, in which the continuation value following defection is $(1-\eta)u^b/(1-\delta) + \eta V(\mathbf{g}, r; \delta)$ rather than $u^b/(1-\delta)$, since a country's defection from the negotiated emission levels goes undetected with probability η.

The compliance constraint at the investment policy stage is equal to:

$$V(\mathbf{g}, r; \delta) \geq c(n-1)(g^b - g_\eta(g, r)) + u^b + \delta((1-\eta^2)\frac{u^b}{1-\delta} + \eta^2 V(\mathbf{g}, r; \delta)), \tag{B.1}$$

where $1 - \eta^2$ is the probability of detecting a country's deviation from either an emission policy or an investment policy that actually occurred. As in the case without uncertainty, a country that deviates from the investment policy by choosing r^b will also deviate from the emission policy by choosing g^b. If a country defects at the investment policy stage, emissions in each non-deviating country are given by:

$$g_\eta(g, r) = \eta g + (1-\eta) g_b(r), \tag{B.2}$$

where $g_b(r) = \overline{y} - c/b - r$, since non-deviating countries comply with the emission level g with some probability, even after one country deviates from its investment policy. Under price regulation, r in (B.2) is replaced by $\widetilde{r} = (\tau^b + \varsigma)/k$.

stark signal technology permits a simple characterization of the results, which are however robust to a more general signal structure, such as the one considered in Green and Porter (1984) in which punishment is triggered with some probability only when a continuous signal falls below a certain threshold. The signal structure adopted in this paper is however similar to that used in Ederington (2003) and Maggi (1999).

A Folk Theorem. As without uncertainty, the first best can be sustained when the discount factor is higher than a critical threshold, which are explicitly characterized in the following lemma.

LEMMA B.1: *The compliance constraint binds at the* s-stage, $s \in \{g, r\}$, *for policy instrument* $\iota \in \{p, q\}$, *when* $\delta < \overline{\delta}^{s}_{\iota}$, *where:*

$$\overline{\delta}^{g}_{q} = \frac{k}{(1-\eta)\,b + (2-\eta)\,k} \quad \textit{and} \quad \overline{\delta}^{r}_{q} = \frac{b - (1-2\eta)\,k}{(2-\eta^2)\,b + \eta\,(2-\eta)\,k},$$

and

$$\overline{\delta}^{g}_{p} = \overline{\delta}^{g}_{q} \quad \textit{and} \quad \overline{\delta}^{r}_{p} = \frac{2\eta - 1}{\eta\,(2-\eta)}.$$

The threshold levels of the discount factor reported in Lemma B.1 include those for the case without uncertainty, as a particular case, when $\eta = 0$. In particular, $\overline{\delta}^{r}_{p}$ tends to minus infinity in that extreme case, since the compliance constraint at the investment policy stage never binds first under price regulation. In contrast, under uncertainty, countries may have a stronger temptation to deviate at the investment policy stage than at the emission policy stage under both quantity regulation and price regulation if domestic policies are not sufficiently transparent: that is, $\overline{\delta}^{r}_{q} > \overline{\delta}^{g}_{q}$ when $\eta > \eta_q \equiv 2 - b/k$ (which requires $k/b < 1/2$) and $\overline{\delta}^{r}_{p} > \overline{\delta}^{g}_{p}$ when $\eta > \eta_p \equiv (b + 2k) / (2b + k)$. To see this, notice that the effect of uncertainty is to induce non-deviating countries to respond less promptly to deviations that occur at the investment policy stage. From (B.2), we can indeed see that $g_{\eta}(\cdot) < g_{b}(\cdot)$. This weaker reaction increases the temptation to defect at the investment policy stage. Hence, satisfying constraint (B.1) may become harder than satisfying compliance with emission policies also under price regulation if policy transparency is low.

Nevertheless, Lemma B.1 shows that $\max\{\overline{\delta}^{g}_{q}, \overline{\delta}^{r}_{q}\} \geq \max\{\overline{\delta}^{g}_{p}, \overline{\delta}^{r}_{p}\}$ for any η. The intuition behind this result is that the non-deviating countries' willingness to raise emissions following a country's defection at the investment policy stage is less under quantity regulation than under price regulation: for a fixed g, $g_{\eta}(g, \widetilde{r}) > g_{\eta}(g, r)$ since $g_{b}(\widetilde{r}) > g_{b}(r)$. Hence, even under uncertainty, a price agreement is able to sustain the first-best outcome for a wider range of δ than a quantity agreement, which makes price instruments superior to quantity controls.

The Optimal Price and Quantity Agreements. The following proposition characterizes the optimal self-enforcing agreements under uncertainty when the global first-best climate treaty cannot be sustained because $\delta < \max\{\overline{\delta}^{g}_{j}, \overline{\delta}^{r}_{j}\}$.

PROPOSITION B.1:

The optimal price agreement is characterized by:

P.i. *If* $\delta \in (\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma}), \overline{\delta}_p^g]$, *which requires* $\eta < \eta_p$,

$$\begin{aligned} \tau &= \tau^* - \Lambda_p^\eta(\delta) \quad \text{ and } \quad \varsigma = \Lambda_p^\eta(\delta), \text{ where} \\ \Lambda_p^\eta(\delta) &= \frac{c(n-1)}{1-\eta\delta}\left(1-\delta-\sqrt{\frac{\delta(1-\eta)((1-\eta\delta)b+(1-\eta)\delta k)}{k}}\right); \end{aligned}$$

P.ii. *If* $\delta \in (\delta_p^g(\boldsymbol{\tau}, \boldsymbol{\varsigma}), \overline{\delta}_p^r]$, *which requires* $\eta > \eta_p$,

$$\begin{aligned} \tau &= \tau^* - \Xi_p^\eta(\delta) \quad \text{ and } \quad \varsigma = -\frac{(1-\eta)(b+k)}{\eta k-(1-\eta)b}\Xi_p^\eta(\delta), \text{ where} \\ \Xi_p^\eta(\delta) &= \frac{c(n-1)(\eta k-(1-\eta)b)}{(1-\eta^2\delta)k}\left(1-\delta-(1-\eta)\sqrt{\frac{(1+\eta\delta)^2k+(1-\delta)^2b}{\eta^2k+(1-\eta)^2b}}\right). \end{aligned}$$

The optimal quantity agreement is characterized by:

Q.i. *Suppose* $k/b > 1/2$. *If* $\delta \in (\delta_q^r(\mathbf{g}, \mathbf{r}), \overline{\delta}_q^g]$, *which requires* $\eta < \eta_q$,

$$g = g^* + \Lambda_q^\eta(\delta) \quad \text{ and } \quad r = r^*, \text{ where } \Lambda_q^\eta(\delta) = \frac{1}{b}\Lambda_p^\eta(\delta);$$

Q.ii. *Suppose* $k/b < 1/2$. *If* $\delta \in (\delta_q^g(\mathbf{g}, \mathbf{r}), \overline{\delta}_q^r]$, *which requires* $\eta > \eta_q$,

$$\begin{aligned} g &= g^* + \Xi_q^\eta(\delta) \quad \text{ and } \quad r = r^* - \frac{b}{b+\eta k}\Xi_q^\eta(\delta), \text{ where} \\ \Xi_q^\eta(\delta) &= \frac{c(n-1)(b+\eta k)}{(1-\eta^2\delta)bk}\left(1-\delta-\sqrt{\frac{((1-\eta^2)\delta)^2b+((1-\eta)(1+\delta\eta))^2k}{b+\eta^2k}}\right). \end{aligned}$$

Overinvestment in Green Technology. Parts (P.i) and (Q.i) of the proposition show the optimal allocation when uncertainty is sufficiently small that the compliance constraint at the emission policy stage binds first under both types of agreement. The optimal sustained emission and investment levels are the same whether governments use quotas or price instruments, as in the case without uncertainty. As δ becomes smaller, emissions must be allowed to increase in order to motivate compliance, but the investment level stays unchanged at the first-best level. To induce countries to invest more than what they find optimal, conditional on the level of emissions, a quantity agreement requires to raise investment mandates, while a price agreement requires investment subsidies.

Punishing Investments. Parts (P.ii) and (Q.ii) show how the optimal agreement must change when uncertainty is high. In this case, the constraint at the investment policy stage binds first when δ decreases under both types of treaty. Compliance with the agreement then requires punishing countries that invest too much in green technology in order to dampen the temptation to defect at the investment policy stage. In a price agreement, investments in green technology must therefore be taxed rather than subsidized.

COROLLARY B.1: *When η is large, the optimal self-enforcing price agreement requires a tax on investment in green technology, rather than a subsidy.*

B.II. Type I Errors

We now consider the scenario in which $\lambda > 0$ and $\eta = 0$. In this case, the punishment *can* occasionally be triggered by mistake along the equilibrium path. Therefore, it may not be optimal to maintain a punishment phase forever. Instead, the duration of the punishment phase, T, should be as short as is necessary to motivate compliance.

Compliance Constraints. A country's intertemporal value of cooperation declines as λ increases:

$$V(\mathbf{g},\mathbf{r};\delta) = u(\mathbf{g},r) - c(n-1)[g_\lambda(g,r) - g] + \delta((1-(1-\lambda)^2)V^P(\mathbf{g},\mathbf{r};\delta) + (1-\lambda)^2 V(\mathbf{g},\mathbf{r};\delta)), \tag{B.3}$$

where $1-(1-\lambda)^2$ is the probability of detecting a country's deviation from either the emission level or the investment level, even if no deviation actually takes place, and $V^P(\mathbf{g},\mathbf{r};\delta)$ is the discounted value of punishment, given by:

$$V^P(\mathbf{g},\mathbf{r};\delta) = \frac{1-\delta^T}{1-\delta}u^b + \delta^T V(\mathbf{g},\mathbf{r};\delta).$$

Even if no country has deviated at the investment policy stage, all countries can erroneously observe a signal that is consistent with a deviation, which occurs with probability λ, and in response each of them will thereafter emit $g_b(r)$. The expected emissions generated by the other countries are therefore:

$$g_\lambda(g,r) = (1-\lambda)g + \lambda g_b(r), \tag{B.4}$$

where $r = \widetilde{r} = (\tau^b + \varsigma)/k$, under price regulation.

For a country to comply with the emission policy g, conditional on upfront investment r, rather than deviating to $g_b(r)$, the following compliance constraint at the emission policy stage must be satisfied:

$$V(\mathbf{g},\mathbf{r};\delta) \geq -\frac{b}{2}(\overline{y} - [g_b(r) + r])^2 - c[g_b(r) + (n-1)g_\lambda(g,r)] - \frac{k}{2}r^2 + \delta V^P(\mathbf{g},\mathbf{r};\delta).$$

The compliance constraint at the investment policy stage is equivalent to that reported in the paper, in which the continuation value $u^b/(1-\delta)$ is replaced by the larger value $u^b + \delta V^P(\mathbf{g}, \mathbf{r}; \delta)$.

Impossibility of a Folk Theorem. An important implication of the existence of type I errors is that a folk theorem does not apply, even for a high discount factor. The first-best outcome would be achievable by requiring a long punishment duration, which is, however, not optimal, since retaliation by non-deviating countries takes place along the equilibrium path and, in turn, reduces the achieved welfare. Hence, punishment must be finite when δ is high and its duration must balance the incentives to comply with either emissions or investments, depending on the stage at which countries have the strongest temptation to deviate.

In a price agreement, it is always more tempting to defect at the emission policy stage than at the investment policy stage. The absence of type II errors implies that deviations at the investment policy stage are immediately detected and harshly punished by non-deviating governments—thanks to the reduced investment in green technology made by the private sector in each country in anticipation of the cooperation breakdown. Although the compliance constraint at the investment policy stage cannot bind first, both compliance constraints can simultaneously bind when uncertainty is high ($\lambda > \lambda_p^g$).

In a quantity agreement, the temptation to defect is stronger at the emission policy stage when $\lambda < \lambda_q^g \equiv 1 - (b/2k)^{1/4}$ (which requires that $k/b > 1/2$), while it is more tempting to deviate at the investment policy stage when $\lambda < \lambda_q^r \equiv 1 - (2k/b)^{1/2}$ (which requires that $k/b < 1/2$). It follows that the constraints simultaneously bind when $\lambda > \max\left\{\lambda_q^g, \lambda_q^r\right\}$.

All binding constraints can be slackened by increasing T. Hence, when the discount factor decreases, the punishment's duration can be increased in order to restore the incentives to comply. We can then determine the critical levels of δ, above which the most cooperative climate policies can be sustained by the prospect of a punishment of finite length, when the binding constraint is at either the emission policy stage or the investment policy stage. When δ falls below these critical levels, T needs to be infinite. The threshold levels of the discount factor generalize those characterized in the baseline model when $\lambda > 0$.

LEMMA B.2: *The compliance constraint binds at the* s-stage, $s \in \{g, r\}$, *for policy instrument* $\iota \in \{p, q\}$, *and the punishment duration is* $T = \infty$, *when* $\delta < \bar{\delta}_\iota^s$, *where:*

$$\bar{\delta}_q^g = \frac{(1-\lambda)^4 k}{b + 2(1-\lambda)^4 k} \quad \text{and} \quad \bar{\delta}_q^r = \frac{(1-\lambda)^2 b - k}{2(1-\lambda)^2 b},$$

and

$$\bar{\delta}_p^g = \frac{((1-\lambda)(\lambda b - (1-\lambda)k))^2}{bk + 2((1-\lambda)(\lambda b - (1-\lambda)k))^2}.$$

The Optimal Price and Quantity Agreements. The following proposition characterizes the optimal self-enforcing agreements and the optimal length of punishment when T is finite (since δ is larger than the critical discount factors) and compliance constraints do not simultaneously bind (since λ is small).[5]

PROPOSITION B.2
In the optimal price agreement, the optimal duration of punishment T and the optimal policies are characterized by:

P. *If $\delta \in [\overline{\delta}_p^g, 1)$, which requires $\lambda < \lambda_p^g$,*

$$T = (\log(\delta))^{-1} \log \left(\frac{\delta bk - (1-2\delta)(1-\lambda)^2 (\lambda b - (1-\lambda) k)^2}{bk + (1-\lambda)^2 (\lambda b - (1-\lambda) k)^2} \right) - 1,$$

$$\tau = \tau^* - \frac{c\,(n-1)\left((1-\lambda)^2 \lambda b + (1-(1-\lambda)^3)\, k\right)}{k} \quad \textit{and} \quad \varsigma = \tau^* - \tau.$$

In the optimal quantity agreement, the optimal duration of punishment T and the optimal policies are characterized by:

Q.i. *Suppose $k/b > 1/2$. If $\delta \in [\overline{\delta}_q^g, 1)$, which requires $\lambda < \lambda_q^g$,*

$$T = (\log(\delta))^{-1} \log \left(\frac{\delta b - (1-2\delta)(1-\lambda)^4 k}{b + (1-\lambda)^4 k} \right) - 1,$$

$$g = g^* + \frac{c\,(n-1)\,(1-(1-\lambda)^3)}{b} \quad \textit{and} \quad r = r^*;$$

Q.ii. *Suppose $k/b < 1/2$. If $\delta \in [\overline{\delta}_q^r, 1)$, which requires $\lambda < \lambda_q^r$,*

$$T = (\log(\delta))^{-1} \log \left(\frac{k - (1-\lambda)^2 (1-2\delta) b}{(1-\lambda)^2 b + k} \right) - 1,$$

$$g = g^* + \frac{c(n-1)\left(\lambda k + (1-(1-\lambda)^2)\, b\right)}{bk} \quad \textit{and} \quad r = r^* - \frac{c(n-1)\,(1-(1-\lambda)^2)}{k}.$$

The Optimal Price Agreement. Part (P) of the proposition describes the optimal self-enforcing agreement when governments use price instruments. Since the compliance constraint at the emission policy stage necessarily binds for any δ, the most cooperative outcome involves a carbon tax which is lower than the optimal Pigouvian tax, in order to mitigate countries' temptation to deviate from the emission policy, and an investment

[5] The characterization of the optimal self-enforcing agreement when T is infinite is provided in the proof.

subsidy in order to induce firms to invest in green technology at the first-best level—which is more than what they find optimal, conditional on emissions.

The Optimal Quantity Agreement. Parts (Q.i) and (Q.ii) describe the optimal self-enforcing agreement when governments use quantity controls. When the compliance constraint at the emission policy stage binds first, emission caps must allow countries to pollute more than in the first best and investment mandates must be set at the first-best level. In contrast, when compliance with investment is more difficult, more emissions must be permitted, while more investment in green technology must be punished.

Welfare. In the case of type I errors, it is no longer true that a price agreement is always better than a quantity agreement. In particular, quota controls can achieve higher welfare than price instruments when the compliance constraint at the emission policy stage is the hardest to satisfy under both types of regulations, i.e., $\lambda < \lambda_q^g$, which requires that $k/b > 1/2$.[6]

By inspecting parts (P) and (Q.i.), we can verify that—although investment is at the first-best level under both types of agreement—price instruments permit countries to emit a level $g = g^* + c\,(n-1)\left((\lambda(1-\lambda)^2/k) + \left(\left(1-(1-\lambda)^3\right)/b\right)\right)$, which is larger than the level permitted under quantity controls. To see the reason behind this, suppose that in a price agreement, emissions are required to be less than the implemented level g, say $g' < g$ (for example, g' is equal to the level implemented in a quantity agreement), but suppose also that investment is still forced to stay at the first-best level. To implement this allocation, the carbon tax should be set higher, say at $\tau' > \tau$, and the investment subsidy lower, say at $\varsigma' < \varsigma$, since technology is expensive in the case here considered, namely $k/b > 1/2$. Given these policies, firms in non-deviating countries adjust their investment to a level of $\widetilde{r}' = (\tau^b + \varsigma')/k < \widetilde{r} = (\tau^b + \varsigma)/k$ if they mistakenly observe a country deviating at the investment policy stage. This lower level of investment allows governments to punish deviations by means of higher emission levels, i.e., $g_b\left(\widetilde{r}'\right) > g_b\left(\widetilde{r}\right)$, even if deviations have not taken place. Overall, each country faces a larger welfare loss because of unduly strong punishments at the emission policy stage following mistakenly observed deviation at the investment policy stage. Since more emissions are allowed in a price agreement than in quantity agreement, the temptation to defect at the emission policy stage is also lower in the former case compared to the latter. It follows that the optimal duration of punishment, which balances out the incentives to comply, is shorter when governments set price instruments rather than quantity mandates. We can however show that a quantity agreement is preferable over a price agreement when technology is expensive and there is uncertainty.

[6] Clearly, when countries must be punished for overinvestment in a quantity agreement—because they are more tempted to deviate at the investment policy stage, which requires that $k/b < 1/2$—while the same countries would not have been punished in a price agreement, price instruments are preferable over quantity controls.

COROLLARY B.2: *When λ is small and technology is expensive, the quantity agreement Pareto dominates the price agreement because, with quantity, firms cannot mistakenly adjust their investments if all countries cooperate.*

The welfare gain associated with a quantity agreement relative to price agreement amounts to $c^2(n-1)^2(1-\lambda)^2\lambda(2k(1-\lambda)-b\lambda)/2k^2$, a level that tends to zero as λ approaches zero, which reconciles with the result on the welfare equivalence between the two type of agreements highlighted in the baseline model when technology is expensive.

The superiority of price instruments over quotas, established in Corollary 2 in the paper, relies on the observation that allowing firms to flexibly adjust their investment decisions to an anticipated reversion to BAU—following another country's actual defection—permits governments to use stronger punishment in a price agreement than in a quantity agreement and, in turn, to obtain more cooperation. The same logic applies in the case with imperfect monitoring, but since uncertainty implies that the stronger punishment enforceable in a price agreement can be triggered when it should not be, the welfare implication can be the opposite of that in the case without uncertainty.

MATHEMATICAL APPENDIX B.

PROOF OF LEMMA B.1:

The proof follows from solving simultaneously the compliance constraint at the emission policy stage and the compliance constraint at the investment policy stage satisfied with equality with respect to δ, when $g = g^*$ and $r = r^*$ under quantity regulation and $\tau = \tau^*$ and $\varsigma = \varsigma^*$ in a price agreement.

PROOF OF PROPOSITION B.1:

First, consider a price agreement when $\delta < \max\left\{\overline{\delta}_p^r, \overline{\delta}_p^g\right\}$. Part (P.i) holds when $\eta < \eta_p \equiv (b+2k)/(2b+k)$. In this case, $\overline{\delta}_p^g > \overline{\delta}_p^r$. For smaller δ that are close enough to $\overline{\delta}_p^g$, the optimization problem is:

$$V_p^\eta(\delta) \equiv \max_{\boldsymbol{\tau},\boldsymbol{\varsigma}} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = \max_{\tau,\varsigma} \frac{1}{1-\delta}\left(-\frac{b}{2}\left(\overline{y}-\left[g(\tau,\varsigma)+r(\tau,\varsigma)\right]\right)^2 - cng(\tau,\varsigma) - \frac{k}{2}r(\tau,\varsigma)^2\right), \tag{B.5}$$

subject to $g(\tau,\varsigma) = \overline{y} - (b\varsigma + (b+k)\tau)/bk$ and $r(\tau,\varsigma) = (\varsigma+\tau)/k$, and

$$\begin{aligned} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) &\geq V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \\ &= \frac{1}{1-\delta\eta}\left(-\frac{b}{2}\left(\overline{y}-\left[g_b(\tau,\varsigma)+r(\tau,\varsigma)\right]\right)^2 - c\left[(n-1)g(\tau,\varsigma)+g_b(\tau,\varsigma)\right] - \frac{k}{2}(\tau,\varsigma)^2\right), \\ &\quad + \frac{1}{1-\delta\eta}\frac{\delta(1-\eta)u^b}{1-\delta} \end{aligned}$$

where $g_b(\tau,\varsigma) = \overline{y} - r(\tau,\varsigma) - c/b$. Solving for ς, we obtain $\varsigma = \tau^* - \tau$. Replacing the equilibrium ς into $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ and solving for τ, we obtain:

$$\tau = \tau^* - \Lambda_p^\eta(\delta,\eta)\,, \text{ where } \Lambda_p^\eta(\delta,\eta) = \frac{c(n-1)}{1-\eta\delta}\left(1-\delta-\sqrt{\frac{\delta(1-\eta)((1-\eta\delta)b+(1-\eta)\delta k)}{k}}\right),$$

and $\partial\Lambda_p^\eta(\delta,\eta)/\partial\delta < 0$ and $\partial\Lambda_p^\eta(\delta,\eta)/\partial\eta > 0$. Hence, $\varsigma = \Lambda_p^\eta(\delta,\eta)$. Part (P.ii) holds when $\eta > \eta_p$. In this case, $\overline{\delta}_p^g < \overline{\delta}_p^r$. For smaller δ that are close enough to $\overline{\delta}_p^r$, the optimization problem consists in maximizing (B.5), subject to:

$$V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \geq V^r(\boldsymbol{\tau_{-i}},\boldsymbol{\varsigma_{-i}};\delta) = \frac{c(n-1)}{1-\delta\eta^2}\left(g^b - g_\eta(g,\widetilde{r})\right) + \frac{u^b}{1-\delta},$$

where $g_\eta(g,\widetilde{r}) = \eta g(\tau,\varsigma) + (1-\eta)\,g_b(\widetilde{r})$, $g_b(\widetilde{r}) = \overline{y} - \widetilde{r} - c/b$, and $\widetilde{r} = (c+\varsigma)/k$. Solving for ς, we obtain $\varsigma = -((1-\eta)(k+b)/(\eta k-(1-\eta)b))(\tau^*-\tau)$. Replacing the equilibrium ς into $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = V^r(\boldsymbol{\tau_{-i}},\boldsymbol{\varsigma_{-i}};\delta)$ and solving for τ, we obtain:

$$\begin{aligned}\tau &= \tau^* - \Xi_p^\eta(\delta,\eta)\,, \text{ where}\\ \Xi_p^\eta(\delta,\eta) &= \frac{c(n-1)(\eta k-(1-\eta)b)}{(1-\eta^2\delta)k}\left(1-\delta-(1-\eta)\sqrt{\frac{(1+\delta\eta)^2k+(1-\delta)^2b}{\eta^2k+(1-\eta)^2b}}\right),\end{aligned}$$

and $\partial\Xi_p^\eta(\delta,\eta)/\partial\delta < 0$ and $\partial\Xi_p^\eta(\delta,\eta)/\partial\eta > 0$. Hence,

$$\varsigma = -\frac{(1-\eta)(k+b)}{\eta k-(1-\eta)b}\Xi_p^\eta(\delta,\eta)\,.$$

Next, consider a quantity agreement when $\delta < \max\left\{\overline{\delta}_q^r, \overline{\delta}_q^g\right\}$. Part (Q.i) holds when $\eta < \eta_q \equiv 2 - b/k$. In this case, $\overline{\delta}_q^g > \overline{\delta}_q^r$. For smaller δ that are close enough to $\overline{\delta}_q^g$, the optimization problem becomes:

$$V_q^\eta(\delta) \equiv \max_{\mathbf{g},r} V(\mathbf{g},r;\delta) = \max_{g,r}\frac{1}{1-\delta}\left(-\frac{b}{2}(\overline{y}-[g+r])^2 - cng - \frac{k}{2}r^2\right), \tag{B.6}$$

subject to:

$$\begin{aligned}V(\mathbf{g},r;\delta) &\geq V^g(\{g_b(r),\mathbf{g}_{-i}\},r;\delta)\\ &= \frac{1}{1-\delta\eta}\left(-\frac{b}{2}(\overline{y}-[g_b(r)+r])^2 - c[g_b(r)+(n-1)g] - \frac{k}{2}r^2 + \frac{\delta(1-\eta)u^b}{1-\delta}\right),\end{aligned}$$

where $g_b(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = r^*$. Substituting the equilibrium r into $V(\mathbf{g},r;\delta) = V^g(\{g_b(r),\mathbf{g}_{-i}\},r;\delta)$ and solving for g, we obtain:

$$g = g^* + \Lambda_q^\eta(\delta,\eta)\,, \text{ where } \Lambda_q^\eta(\delta,\eta) = \frac{1}{b}\Lambda_p^\eta(\delta,\eta)\,,$$

$\partial\Lambda_q^\eta(\delta,\eta)/\partial\delta<0$ and $\partial\Lambda_q^\eta(\delta,\eta)/\partial\eta>0$. Part (Q.ii) holds when $\eta>\eta_q$. In this case, $\overline{\delta}_q^g<\overline{\delta}_q^r$. For smaller δ that are close enough to $\overline{\delta}_q^r$, the optimization problem consists in maximizing (B.6), subject to:

$$V(\mathbf{g},r;\delta)\geq V^r(\mathbf{g}_{-i},\mathbf{r}_{-i};\delta)=\frac{c(n-1)}{1-\delta\eta^2}\left(g^b-g_\eta(g,r)\right)+\frac{u^b}{1-\delta},$$

where $g_\eta(g,r)=\eta g+(1-\eta)g_b(r)$ and $g_b(r)=\overline{y}-r-c/b$. Solving for r, we obtain $r=(b/(b+\eta k))(\overline{y}-g)-(1-\eta)cn/(b+\eta k)$. Substituting the equilibrium r into $V(\mathbf{g},r;\delta)=V^r(\mathbf{g}_{-i},\mathbf{r}_{-i};\delta)$ and solving for g, we obtain:

$$\begin{aligned}&g=g^*+\Xi_q^\eta(\delta,\eta)\text{, where}\\ &\Xi_q^\eta(\delta,\eta)=\frac{c(n-1)(b+\eta k)}{(1-\eta^2\delta)bk}\left(1-\delta-\sqrt{\frac{((1-\eta^2)\delta)^2b+((1-\eta)(1+\delta\eta))^2k}{b+\eta^2k}}\right),\end{aligned}$$

$\partial\Xi_q^\eta(\delta,\eta)/\partial\delta<0$ and $\partial\Xi_q^\eta(\delta,\eta)/\partial\eta>0$. Hence,

$$r=r^*-\frac{b}{b+\eta k}\Xi_q^\eta(\delta,\eta).$$

PROOF OF LEMMA B.2 AND PROPOSITION B.2:

First, consider a price agreement. To demonstrate part (P), assume that the compliance constraint at the investment policy stage is slack, while the compliance constraint at the emission policy stage binds. In this case, the optimal agreement is obtained by solving the following maximization problem:

$$\begin{aligned}V_p^\lambda(\delta)&\equiv\max_{\boldsymbol{\tau},\boldsymbol{\varsigma}}V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)\\ &=\max_{\tau,\varsigma}\frac{1}{1-\delta}\begin{pmatrix}-\frac{b}{2}(\overline{y}-[g(\tau,\varsigma)+r(\tau,\varsigma)])^2-cng(\tau,\varsigma)-\frac{k}{2}r(\tau,\varsigma)^2\\ -c(n-1)[g_\lambda(g,\widetilde{r})-g(\tau,\varsigma)]-\frac{1-(1-\lambda)^2}{(1-\lambda)^2}\psi_g(\tau,\varsigma)\end{pmatrix},\end{aligned}$$

where $\psi_g(\tau,\varsigma)\equiv(c-b(\overline{y}-[g(\tau,\varsigma)+r(\tau,\varsigma)]))^2/2b$, $g(\tau,\varsigma)=\overline{y}-(b\varsigma+(b+k)\tau)/bk$, $r(\tau,\varsigma)=(\varsigma+\tau)/k$, $g_\lambda(g,\widetilde{r})=(1-\lambda)g(\tau,\varsigma)+\lambda(\overline{y}-\widetilde{r}-c/b)$, and $\widetilde{r}=(c+\varsigma)/k$. Solving for τ and ς, we obtain:

$$\tau=\tau^*-\frac{c(n-1)((1-\lambda)^2\lambda b+(1-(1-\lambda)^3)k)}{k}\quad\text{and}\quad\varsigma=\tau^*-\tau,\tag{B.7}$$

which implements the allocation:

$$g=g^*+c(n-1)\left(\frac{\lambda(1-\lambda)^2}{k}+\frac{1-(1-\lambda)^3}{b}\right)\quad\text{and}\quad r=r^*.$$

Combining $V^P(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = \left((1-\delta^T)/(1-\delta)\right)u^b + \delta^T V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ and $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) - V^P(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = (1/(\delta(1-\lambda)^2))\,\psi_g(\tau,\varsigma)$ and using (B.7), we obtain:

$$T = [\log(\delta)]^{-1}\log(\Gamma_p^g(\delta)) - 1, \tag{B.8}$$

where $\Gamma_p^g(\delta) \equiv ((1-\delta)\,bk)\,/(bk+(1-\lambda)^2(\lambda b-(1-\lambda)\,k)^2)+2\delta-1$. By solving $\Gamma_p^g(\delta)=0$ with respect to δ, we obtain:

$$\bar{\delta}_p^g = \frac{((1-\lambda)(\lambda b-(1-\lambda)\,k))^2}{bk+2\,((1-\lambda)(\lambda b-(1-\lambda)\,k))^2}.$$

We next show the condition under which compliance constraint at the investment policy stage is satisfied when climate policies are (B.7) and the length of punishment is (B.8), by verifying that:

$$V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) - V^P(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \geq \frac{\psi_r(\tau,\varsigma)}{\delta(1-\lambda)^2}, \tag{B.9}$$

where $\psi_r(\tau,\varsigma) \equiv c\,(n-1)\left(g^b - g_b(\widetilde{r})\right) - \left(u(g,r)-u^b\right)$, $g_b(\widetilde{r}) = \overline{y}-\widetilde{r}-c/b$, and $\widetilde{r} = (c+\varsigma)/k$. Inequality (B.9) is satisfied when $\lambda \leq \lambda_p^g$, where λ_p^g is implicitly determined from (B.9). It follows that both compliance constraints at the emission and investment policy stages simultaneously bind when $\lambda > \lambda_p^g$. When $\lambda < \lambda_p^g$, the compliance constraint at the emission policy stage is binding. For $\delta \leq \bar{\delta}_p^g$, $T=\infty$ and the optimal agreement is obtained by $\max_{\boldsymbol{\tau},\boldsymbol{\varsigma}} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$, where $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ is given by:

$$\begin{aligned}&-\frac{1}{1-\delta(1-\lambda)^2}\left(\frac{b}{2}\left(\overline{y}-[g(\tau,\varsigma)+r(\tau,\varsigma)]\right)^2 + c\,[g(\tau,\varsigma)+(n-1)\,g_\lambda(g,\widetilde{r})] + \frac{k}{2}r(\tau,\varsigma)^2\right)\\ &+\frac{1-(1-\lambda)^2}{1-\delta(1-\lambda)^2}\frac{\delta u^b}{1-\delta},\end{aligned}$$

subject to $g(\tau,\varsigma) = \overline{y}-(b\varsigma+(b+k)\tau)/bk$ and $r(\tau,\varsigma) = (\varsigma+\tau)/k$, and

$$\begin{aligned}V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) &\geq V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)\\ &= -\left(\frac{b}{2}\left(\overline{y}-[g_b(\tau,\varsigma)+r(\tau,\varsigma)]\right)^2 + c\,[g_b(\tau,\varsigma)+(n-1)\,g_\lambda(g,\widetilde{r})] + \frac{k}{2}r(\tau,\varsigma)^2\right) + \frac{\delta u^b}{1-\delta}.\end{aligned}$$

where $g_b(\tau,\varsigma) = \overline{y}-c/b-r(\tau,\varsigma)$, $g_\lambda(g,\widetilde{r}) = (1-\lambda)\,g(\tau,\varsigma)+\lambda g_b(\widetilde{r})$, $g_b(\widetilde{r}) = \overline{y}-\widetilde{r}-c/b$, and $\widetilde{r} = (c+\varsigma)/k$. Solving for ς, we obtain $\varsigma = \tau^* - \tau$. Substituting the equilibrium ς into $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ and solving for τ, we obtain:

$$\begin{aligned}\tau &= \tau^g - \frac{(1-\delta)c(n-1)(1-\lambda)^2(\lambda b-(1-\lambda)\,k)}{k}\\ &\quad -\frac{(1-\lambda)\,c(n-1)}{k}\sqrt{\delta(\delta(1-\lambda)^2((1-\lambda)^2k^2+\lambda^2b^2)+bk(1-2\lambda(1-\lambda)^3\delta))}\end{aligned}$$

where $\tau^g = \tau^* - c\,(n-1)\left((1-\lambda)^2\lambda b + (1-(1-\lambda)^3)k\right)/k$.

Next, consider a quantity agreement. To show part (Q.i), assume that the compliance constraint at the investment policy stage is slack, while the compliance constraint at the emission policy stage binds. In this case, the optimal climate policy is obtained by solving the following maximization problem:

$$V_q^\lambda(\delta) \equiv \max_{\mathbf{g},\mathbf{r}} V(\mathbf{g},\mathbf{r};\delta) = \max_{g,r} \frac{1}{1-\delta}\left(\begin{array}{c} -\frac{b}{2}\left(\overline{y}-[g+r]\right)^2 - cng - \frac{k}{2}r^2 \\ -c\,(n-1)\left[g_\lambda(g,r)-g\right] - \frac{1-(1-\lambda)^2}{(1-\lambda)^2}\psi_g(g,r) \end{array}\right),$$

where $\psi_g(g,r) \equiv \left(c - b\left(\overline{y}-(g+r)\right)\right)^2/2b$ and $g_\lambda(g,r) = (1-\lambda)\,g + \lambda\left(\overline{y}-r-c/b\right)$. Solving for g and r, we obtain:

$$g = g^* + \frac{c\,(n-1)\left(1-(1-\lambda)^3\right)}{b} \quad \text{and} \quad r = r^*. \tag{B.10}$$

Combining $V^P(\mathbf{g},\mathbf{r};\delta) = \left(\left(1-\delta^T\right)/(1-\delta)\right)u^b + \delta^T V(\mathbf{g},\mathbf{r};\delta)$ and $V(\mathbf{g},\mathbf{r};\delta) - V^P(\mathbf{g},\mathbf{r};\delta) = (1/(\delta(1-\lambda)^2))\psi_g(g,r)$ and using (B.10), we obtain, we obtain:

$$T = [\log(\delta)]^{-1}\log\left(\Gamma_q^g(\delta)\right) - 1, \tag{B.11}$$

where $\Gamma_q^g(\delta) \equiv (\delta b - (1-2\delta)(1-\lambda)^4 k)/(b + (1-\lambda)^4 k)$. By solving $\Gamma_q^g(\delta)$ with respect to δ, we obtain:

$$\bar{\delta}_q^g = \frac{(1-\lambda)^4 k}{b + 2\,(1-\lambda)^4 k}.$$

We next show the condition under which compliance constraint at the investment policy stage is satisfied when climate policies are (B.10) and the length of punishment is (B.11). We verify that:

$$V(\mathbf{g},\mathbf{r};\delta) - V^P(\mathbf{g},\mathbf{r};\delta) > \frac{\psi_r(g,r)}{\delta\,(1-\lambda)^2}, \tag{B.12}$$

where $\psi_r(g,r) \equiv c\,(n-1)\left(g^b - g_b(r)\right) - \left(u(g,r) - u^b\right)$ and $g_b(r) = \overline{y} - r - c/b$. Inequality (B.12) is satisfied when $\lambda < \lambda_q^g \equiv 1 - (b/2k)^{1/4}$, which requires $k/b > 1/2$. To show part (Q.ii), assume that the compliance constraint at the emission policy stage is slack, while the compliance constraint at the investment policy stage binds. In this case, the optimal climate policy is obtained by solving:

$$V_q^\lambda(\delta) \equiv \max_{\mathbf{g},\mathbf{r}} V(\mathbf{g},\mathbf{r};\delta) = \max_{g,r} \frac{1}{1-\delta}\left(\begin{array}{c} -\frac{b}{2}\left(\overline{y}-[g+r]\right)^2 - cng - \frac{k}{2}r^2 \\ -c\,(n-1)\left[g_\lambda(r)-g\right] - \frac{p}{q_r - p}\psi_r(g,r) \end{array}\right),$$

and it is equal to:

$$g = g^* + \frac{c(n-1)((1-(1-\lambda)^2)b + \lambda k)}{bk} \quad \text{and} \quad r = r^* - \frac{c(n-1)(1-(1-\lambda)^2)}{k}. \tag{B.13}$$

Using (B.13), the optimal T is obtained by solving $V(\mathbf{g}, \mathbf{r}; \delta) - (1/(1-\delta))\, u^b = (1/(\delta(1-\delta^T)(1-\lambda)^2))\psi_r(g, r)$, which is equal to:

$$T = [\log(\delta)]^{-1} \log\left(\Gamma_q^r(\delta)\right) - 1, \tag{B.14}$$

where $\Gamma_q^r(\delta) \equiv (k - (1-\lambda)^2(1-2\delta)b)/((1-\lambda)^2 b + k)$. Solving $\Gamma_q^r(\delta)$ with respect to δ, we obtain:

$$\bar{\delta}_q^r = \frac{(1-\lambda)^2 b - k}{2(1-\lambda)^2 b}.$$

We next show under which condition compliance constraint at the emission policy stage is satisfied when climate policies are by (B.13) and the length of punishment is (B.14). We verify that:

$$V(\mathbf{g}, \mathbf{r}; \delta) - V^P(\mathbf{g}, \mathbf{r}; \delta) > \frac{\psi_g(g, r)}{\delta(1-\lambda)^2}. \tag{B.15}$$

Condition (B.15) is satisfied when $\lambda < \lambda_q^r \equiv 1 - (2k/b)^{1/2}$, which requires $k/b < 1/2$. It follows that both compliance constraint at the emission and investment policy stages simultaneously bind when $\lambda > \max\left\{\lambda_q^g, \lambda_q^r\right\}$. When $\lambda < \lambda_q^g$, the compliance constraint at the emission policy stage is binding. For $\delta \le \bar{\delta}_q^g$, $T = \infty$ and the optimal agreement is obtained by $\max_{\mathbf{g},\mathbf{r}} V(\mathbf{g}, \mathbf{r}; \delta)$, where $V(\mathbf{g}, \mathbf{r}; \delta)$ is given by:

$$-\frac{1}{1-\delta(1-\lambda)^2}\left(\frac{b}{2}(\overline{y} - [g+r])^2 + c[g + (n-1)g_\lambda(g, r)] + \frac{k}{2}r^2\right) + \frac{1-(1-\lambda)^2}{1-\delta(1-\lambda)^2}\frac{\delta u^b}{1-\delta},$$

subject to:

$$\begin{aligned} V(\mathbf{g}, \mathbf{r}; \delta) &\geq V^g(\mathbf{g}, \mathbf{r}; \delta) \\ &= -\left(\frac{b}{2}(\overline{y} - [g_b(r) + r])^2 + c[g_b(r) + (n-1)g_\lambda(g, r)] + \frac{k}{2}r^2\right) + \frac{\delta u^b}{1-\delta}. \end{aligned}$$

where $g_\lambda(g, r) = (1-\lambda)g + \lambda(\overline{y} - r - c/b)$ and $g_b(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = r^*$. Substituting the equilibrium r into $V(\mathbf{g}, \mathbf{r}; \delta) = V^g(\mathbf{g}, \mathbf{r}; \delta)$ and solving for g, we obtain:

$$g = g^g + \frac{c(n-1)(1-\lambda)}{b}\left((1-\delta)(1-\lambda)^2 - \sqrt{\frac{\delta(b + \delta k(1-\lambda)^4)}{k}}\right),$$

where $g^g = g^* + c(n-1)(1-(1-\lambda)^3)/b$. When $\lambda < \lambda_q^r$, the compliance constraint at the investment policy stage is binding. For $\delta \leq \bar{\delta}_q^r$, $T = \infty$ and the optimal agreement is obtained by $\max_{\mathbf{g},\mathbf{r}} V(\mathbf{g}, \mathbf{r}; \delta)$, subject to:

$$\begin{aligned} V(\mathbf{g}, \mathbf{r}; \delta) &\geq V^r(\mathbf{r}_{-i}; \delta) \\ &= -\left(\frac{b}{2}\left(\overline{y} - \left[g^b + r^b\right]\right)^2 + c\left[g^b + (n-1)\, g_b(r)\right] + \frac{k}{2}\left(r^b\right)^2\right) + \frac{\delta u^b}{1-\delta}, \end{aligned}$$

where $g_b(r) = \overline{y} - r - c/b$. Solving for r, we obtain:

$$r = (\overline{y} - g) - \frac{c(\lambda + (1-\lambda)\, n)}{b}.$$

Substituting the equilibrium r into $V(\mathbf{g}, \mathbf{r}; \delta) = V^r(\mathbf{r}_{-i}; \delta)$ and solving for g, we obtain:

$$g = g^r + \frac{c(n-1)(1-\lambda)}{k}\left((1-\delta)(1-\lambda) - \sqrt{\frac{(\delta^2(1-\lambda)^2 b + k)}{b}}\right),$$

where $g^r = g^* + c(n-1)((1-(1-\lambda)^2)b + \lambda k)/bk$. Hence,

$$r = r^r - \frac{c(n-1)(1-\lambda)}{k}\left((1-\delta)(1-\lambda) - \sqrt{\frac{(\delta^2(1-\lambda)^2 b + k)}{b}}\right).$$

where $r^r = r^* - c(n-1)(1-(1-\lambda)^2)/k$.

PROOF OF COROLLARY B.2:

When $\lambda < \lambda_q^g$, which requires $k/b > 1/2$, we obtain $V_q^\lambda(\delta) - V_p^\lambda(\delta) = c^2(n-1)^2(1-\lambda)^2\lambda(2k(1-\lambda) - b\lambda)/2k^2 > 0$, where the values $V_q^\lambda(\delta)$ and $V_p^\lambda(\delta)$ are reported in the proof of Proposition B.2.

REFERENCES

Acemoglu, Daron, Akcigit Ufuk, Douglas Hanley, and William Kerr. 2016. "Transition to Clean Technology." *Journal of Political Economy*, 124, 52–104.

Ederington, Josh. 2003. "Policy linkage and uncertainty in international agreements." *Economic Inquiry*, 41, 305–317.

Green, Edward J. and Robert H. Porter. 1984. "Noncooperative Collusion under Imperfect Price Information." *Econometrica*, 52, 87–100.

Harstad, Bård, Francesco Lancia, and Alessia Russo. 2019. "Compliance Technology and Self-enforcing Agreements." *Journal of the European Economic Association*, 19, 1–29.

Maggi, Giovanni. 1999. "The Role of Multilateral Institutions in International Trade Cooperation." *American Economic Review*, 89(1), 190–214.