Supplement to "Public Education and Pensions in Democracy: A Political Economy Theory"*

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Appendix B

This appendix presents some supplementary material. Appendix B.1 provides the political microfoundation of the model. Appendix B.2 extends the analysis to complete markets for student loans. Appendix B.3 presents the numerical strategy in detail. Finally, Appendix B.4 presents the detailed numerical analysis of a financial shock.

B.1 PROBABILISTIC VOTING MODEL

The political equilibrium discussed in the paper is based on the voting model in Lindbeck and Weibull (1987) applied to an overlapping generation environment with intergenerational transfers. The population consists of two groups of voters, i.e. adults and the elderly, denoted by $i \in \{a, o\}$. Any adult group is n times larger than the elderly group. The electoral competition takes place between two office-seeking candidates, denoted by $\iota \in \{\mathcal{L}, \mathcal{R}\}$. Candidates and voters move sequentially. At any t, candidates first announce their political platform $\{\tau_t, f_t, z_t, b_t\}$, subject to the public balanced-budget constraints $\tau_t l_t h_t = n f_t$ and $z_t l_t h_t = (1/n)\, b_t$ with $f_t \geq 0$ and $b_t \geq 0$. The candidates cannot commit to future policies. Second, voter j belonging to

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cohort i chooses a candidate based on her fiscal platform and ideology which is equal to $\sigma_j^i + \eta$. The parameter σ_j^i is an individual-specific shock drawn from a symmetric and cohort-specific uniform distribution in the support $\left[-1/\left(2\boldsymbol{\sigma}^{\mathbf{i}}\right), 1/\left(2\boldsymbol{\sigma}^{\mathbf{i}}\right)\right]$. A value of zero indicates neutrality in terms of ideological bias, while a positive value indicates that individual j prefers the candidate $\mathcal{R}$ over his opponent. Thus, individuals belonging to the same group may still vote differently. The parameter η is an aggregate shock capturing the ex-post average success of candidate $\mathcal{R}$. It is drawn from a uniform i.i.d. distribution on $[-1/(2\boldsymbol{\eta}), 1/(2\boldsymbol{\eta})]$.[1] The realizations of both shocks become known after the political platforms have been announced. With this in mind, an adult voter j prefers candidate $\mathcal{R}$ over $\mathcal{L}$ as long as the idiosyncratic ideological bias, σ_j^a, is larger than the difference in the indirect utility achieved from voting for one of the two candidates, net of an aggregate shock η. Formally,

$$\sigma_j^a \geq \sigma^a\left(\tau_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, b_{t+1}^{\iota}\right) \equiv \mathcal{U}^a\left(\tau_t^{\mathcal{L}}, z_t^{\mathcal{L}}, h_t, b_{t+1}^{\iota}\right) - \mathcal{U}^a\left(\tau_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, b_{t+1}^{\iota}\right) - \eta$$

where $\sigma^a\left(\tau_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, b_{t+1}^{\iota}\right)$ identifies the swing voter within the cohort of adults, who is indifferent between the two candidates. Likewise, an elderly voter j prefers candidate $\mathcal{R}$ over $\mathcal{L}$ if and only if:

$$\sigma_j^o \geq \sigma^o\left(b_t^{\mathcal{L}}, b_t^{\mathcal{R}}, s_{t-1}\right) \equiv \mathcal{U}^o\left(b_t^{\mathcal{L}}, s_{t-1}\right) - \mathcal{U}^o\left(b_t^{\mathcal{R}}, s_{t-1}\right) - \eta$$

Conditional on η, the shares of voters belonging to groups a and o who support candidate $\mathcal{R}$ are respectively:

$$\lambda^a\left(\tau_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, b_{t+1}^{\iota}\right) \equiv (1/2) - \boldsymbol{\sigma}^a \sigma^a\left(\tau_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, b_{t+1}^{\iota}\right)$$

[1]The assumption of uniform distribution is for simplicity (Banks and Duggan, 2005).

and

$$\lambda^{o}\left(b_t^{\mathcal{L}}, b_t^{\mathcal{R}}, s_{t-1}\right) \equiv (1/2) - \boldsymbol{\sigma}^{o}\sigma^{o}\left(b_t^{\mathcal{L}}, b_t^{\mathcal{R}}, s_{t-1}\right)$$

Under a majority rule, candidate $\mathcal{R}$ wins the election if and only if he obtains the largest share of votes, i.e., $\lambda^{a}\left(\tau_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, b_{t+1}^{\iota}\right) + (1/n)\,\lambda^{o}\left(b_t^{\mathcal{L}}, b_t^{\mathcal{R}}, s_{t-1}\right) > (1+1/n)/2$. Note that $\lambda^{a}(\cdot)$ and $\lambda^{a}(\cdot)$ are stochastic variables since η is a random shock. This implies that the probability of $\mathcal{R}$ winning is given by $p^{\mathcal{R}}\left(\tau_t^{\mathcal{L}}, b_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, b_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, s_{t-1}, b_{t+1}^{\iota}\right) \equiv \Pr\left(\eta \geq \tilde{\eta}(\cdot)\right)$ where $\tilde{\eta}\left(\tau_t^{\mathcal{L}}, b_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, b_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, s_{t-1}, b_{t+1}^{\iota}\right)$ is equal to:

$$\frac{\boldsymbol{\sigma}^{o}}{\boldsymbol{\sigma}^{a}+\boldsymbol{\sigma}^{o}}\left(\mathcal{U}^{o}\left(b_t^{\mathcal{L}}, s_{t-1}\right) - \mathcal{U}^{o}\left(b_t^{\mathcal{R}}, s_{t-1}\right)\right) + \frac{n\boldsymbol{\sigma}^{a}}{\boldsymbol{\sigma}^{a}+\boldsymbol{\sigma}^{o}}\left(\mathcal{U}^{a}\left(\tau_t^{\mathcal{L}}, z_t^{\mathcal{L}}, h_t, b_{t+1}^{\iota}\right) - \mathcal{U}^{a}\left(\tau_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, b_{t+1}^{\iota}\right)\right)$$

Hence, candidate $\mathcal{R}$ seeks to maximize the probability of winning the election, namely:

$$\max_{f_t^{\mathcal{R}}, \tau_t^{\mathcal{R}}, b_t^{\mathcal{R}}, z_t^{\mathcal{R}}} (1/2) - \boldsymbol{\eta}\tilde{\eta}\left(\tau_t^{\mathcal{L}}, b_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, b_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, s_{t-1}, b_{t+1}^{\iota}\right) \tag{B.1}$$

Likewise, for candidate $\mathcal{L}$, the objective function is $p^{\mathcal{L}}\left(\tau_t^{\mathcal{L}}, b_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, b_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, s_{t-1}, b_{t+1}^{\iota}\right) \equiv \Pr\left(\eta < \tilde{\eta}(\cdot)\right)$, which implies that her maximization problem is:

$$\max_{f_t^{\mathcal{L}}, \tau_t^{\mathcal{L}}, b_t^{\mathcal{L}}, z_t^{\mathcal{L}}} (1/2) + \boldsymbol{\eta}\tilde{\eta}\left(\tau_t^{\mathcal{L}}, b_t^{\mathcal{L}}, z_t^{\mathcal{L}}, \tau_t^{\mathcal{R}}, b_t^{\mathcal{R}}, z_t^{\mathcal{R}}, h_t, s_{t-1}, b_{t+1}^{\iota}\right)$$

Note that voters do not punish candidates for their past behavior but rather decide in terms of current and future fiscal policies. Given the lack of commitment to future policies, however, the current candidates must decide on the optimal fiscal platform, knowing that it will be replaced by a future policy maker with some probability $p^{\iota}(\cdot)$. In general, this probability is a nontrivial function of the state variable and requires the use of numerical methods for its characterization. However, in a symmetric MPE where candidates are homogenous the probability of reelection is constant and equal to 1/2. To see this, consider a two-period economy and solve by backward induction. In the last period, $t = 2$, adults have no future and therefore $\mathcal{U}^a\left(\tau_2^\iota, z_2^\iota, h_2\right) = \log((1-\tau_2^\iota - z_2^\iota)\, L\left(\tau_2^\iota, z_2^\iota\right) h_2 + F\left(L\left(\tau_2^\iota, z_2^\iota\right)\right) h_2)$. The maximization problem of candidate $\mathcal{R}$, as described in Eq. (B.1), then simplifies to:

$$\max_{f_2^{\mathcal{R}}, \tau_2^{\mathcal{R}}, z_2^{\mathcal{R}}, b_2^{\mathcal{R}}} n\, \mathcal{U}^a\left(\tau_2^{\mathcal{R}}, z_2^{\mathcal{R}}, h_2\right) + \left(\boldsymbol{\sigma}^o / \boldsymbol{\sigma}^a\right)\, \mathcal{U}^o\left(b_2^{\mathcal{R}}, s_1\right)$$

Likewise for candidate $\mathcal{L}$ at $t = 2$:

$$\max_{f_2^{\mathcal{L}}, \tau_2^{\mathcal{L}}, z_2^{\mathcal{L}}, b_2^{\mathcal{L}}} n\, \mathcal{U}^a\left(\tau_2^{\mathcal{L}}, z_2^{\mathcal{L}}, h_2\right) + \left(\boldsymbol{\sigma}^o / \boldsymbol{\sigma}^a\right)\, \mathcal{U}^o\left(b_2^{\mathcal{L}}, s_1\right)$$

This implies that office-seeking candidates propose the same equilibrium platform , namely $\{f_2^\iota, \tau_2^\iota, b_2^\iota, z_2^\iota\} = \{f_2, \tau_2, b_2, z_2\}$ for every ι. It follows that $\tilde{\eta}\left(\tau_2^{\mathcal{L}}, b_2^{\mathcal{L}}, z_2^{\mathcal{L}}, \tau_2^{\mathcal{R}}, b_2^{\mathcal{R}}, z_2^{\mathcal{R}}, h_2, s_1\right) = 0$ and in turn $p^\iota\left(\tau_2^{\mathcal{L}}, b_2^{\mathcal{L}}, z_2^{\mathcal{L}}, \tau_2^{\mathcal{R}}, b_2^{\mathcal{R}}, z_2^{\mathcal{R}}, h_2, s_1\right) = 1/2$. At time $t = 1$, adult voters face a two-period temporal horizon. However, the candidates anticipate that $b_2^{\mathcal{R}} = b_2^{\mathcal{L}} = b_2$ independently of the identity of representative in the subsequent period. It is then straightforward to show that both candidates announce the same political platform $\{f_1, \tau_1, b_1, z_1\}$ since they rationally internalize that the probability of being reelected in the next period is equal to 1/2, independently of current policies. This argument holds for every period t. In summary, the platforms of the two candidates converge in equilibrium to the same fiscal policies, which maximize the weighted sum of the indirect utilities of adults and the elderly, namely:

$$\max_{f_t, \tau_t, z_t, b_t} \mathcal{U}^a\left(\tau_t, z_t, h_t, b_{t+1}\right) + (\phi/n)\, \mathcal{U}^o\left(b_t, s_{t-1}\right)$$

where $\phi \equiv \boldsymbol{\sigma}^o / \boldsymbol{\sigma}^a$ reflects the political power of the elderly relative to adult voters.

B.2 COMPLETE MARKETS

In this appendix, we test the robustness of a PIS to the presence of complete markets for student loans. Consider an economy similar to the one described in Section 4.2, i.e. a small open economy populated by overlapping generations of three-period-lived agents in which policy makers have access to lump-sum taxes. Agents acquire skills in the first period, work and save in the second and retire in the final period. In the absence of market restrictions, an individual born in $t-1$ borrows e_{t-1} units of physical resources to invest in her education when young and repays Re_{t-1} while working in period t. This leads to a level of human capital given by $h_t = H(h_{t-1}, e_{t-1}, f_{t-1}) \equiv Ah_{t-1}^{\theta}(e_{t-1}+f_{t-1})^{1-\theta}$ where h_{t-1} is parental human capital and f_{t-1} is public education investment. As in Glomm and Ravikumar (1992), private and public investment in education are perfect substitutes. Hence, what determines human capital is the sum of the two and not their proportions. Within each period, governments determine public education and pensions expenditure. Following that, agents make their private decisions. The life-cycle optimization problem for an agent born in period $t-1$ is

$$\max_{e_{t-1}, s_t} \log(c_t^a) + \beta \log\left(c_{t+1}^o\right)$$

subject to the per-period budget constraints for adults and the elderly, the borrowing constraint, and the human capital technology:

$$\begin{aligned} c_t^a + s_t + Re_{t-1} &\leq h_t - nf_t - (1/n)\, b_t \\ c_{t+1}^o &\leq Rs_t + b_{t+1} \\ 0 &\leq e_{t-1} \leq h_t/R \\ h_t &= Ah_{t-1}^{\theta}(e_{t-1}+f_{t-1})^{1-\theta} \end{aligned}$$

The individual's optimal choices of private saving and private education are then equal to

$s_t = (\beta/(1+\beta))(h_t - Re_{t-1} - nf_t - (1/n)b_t) - (1/R(1+\beta))b_{t+1}$ and

$$e_{t-1} = \begin{cases} \left(\frac{(1-\theta)A}{R}\right)^{\frac{1}{\theta}} h_{t-1} - f_{t-1} & \text{iff} \quad f_{t-1} < \left(\frac{(1-\theta)A}{R}\right)^{\frac{1}{\theta}} h_{t-1} \\ 0 & \text{iff} \quad f_{t-1} \geq \left(\frac{(1-\theta)A}{R}\right)^{\frac{1}{\theta}} h_{t-1} \end{cases} \tag{B.2}$$

Eq. (B.2) implies that it is optimal for the young to borrow resources against their future income if and only if the level of public education financed by elected governments is below the maximum level of private education. According to Proposition 2, the level of public education investment is equal to $f_{t-1} = \psi h_{t-1}$ in the absence of private education, where ψ is the solution of the following polynomial:

$$\psi = \left(\frac{A}{R}((1-\theta) + n\theta\psi)\right)^{\frac{1}{\theta}}$$

Given $\theta \geq 0$, we have $f_{t-1} \geq \left(\frac{(1-\theta)A}{R}\right)^{\frac{1}{\theta}} h_{t-1}$. Hence, the optimal individual choice of private education is $e_{t-1} = 0$ insofar as education is publicly provided. Proposition 2 then applies.

These findings have important implications for the robustness of the intergenerational mechanism presented in the paper. Remarkably, the analysis remains unaltered if we introduce the possibility of private education. The existence of a political rent enjoyed by retirees can stimulate public education insofar as it garners political support for growth-oriented policies. When the relative political power of the elderly is so weak that the nonnegativity constraint of pension transfers is binding, the government's incentive to invest in public education vanishes. When the possibility of private education does exist, however, the economy avoids converging to a steady state with no human capital if pensions are not financed. In fact, young agents will borrow resources in the private market to accumulate skills. Growth will be sustained in the long run, although it will be slower than that generated by public education investment. This suggests that even when markets are complete, elected governments can improve on the market allocation by providing public education since market institutions can at most react to the short term and fail to internalize intergenerational human capital spillover. In contrast, the prospect of follow-up intergenerational fiscal policies induces governments with short-term mandate to internalize the long-run consequences of human capital investment, as shown in Section 4.2.1.

In this context, it is possible to infer some interesting policy implications regarding pension reform. Consider, for example, a shift from a tax-based, pay-as-you-go system to an investment-based retirement system. In this case, the destruction of the rent opportunities for the elderly in the form of pension transfers wipes out the incentive for governments to invest in public education. Current adults will certainly gain from this reform because they can invest in a private asset whose return is higher than the implicit rate of return in a PIS. However, current elderly will lose since they will receive no pension transfers even though they contributed to the program as adults. Furthermore, a welfare loss may be borne by all future generations if the consumption reduction associated with a weaker growth rate of labor earnings is not compensated for by the gains associated with the higher return delivered through investment in private assets. In this case, this paper also speaks to the current debate on pension reform and highlights the need for the optimal design of social security programs to take into consideration the political economic consequences for human capital formation.

B.3 NUMERICAL ALGORITHM

In this appendix, we present the computational strategy used to characterize an MPE with endogenous labor supply. We need to compute five functions: $\mathcal{F}(h, s_{-})$, $\mathcal{T}(h, s_{-})$, $\mathcal{Z}(h, s_{-})$, $\mathcal{B}(h, s_{-})$, and $\mathcal{A}(f, \tau, z, h|\mathcal{B})$, which solve the first-order conditions (9) and (10) in tandem with the government budget constraints (1) and (2) and the private Euler equation. We follow a standard projection method with n-order Chebyshev polynomials. Within the class of orthogonal polynomials, the Chebyshev method stands out for its efficiency in approximating smooth functions.[2] Even though the application of this method to the problem at hand is quite straightforward, there is a specific feature of the model that turns out to be useful when embedded within the algorithm. Policy rules should be approximated using two-dimensional Chebyshev polynomials, $\theta_{ij}(h, s_-)$, which are the tensor products of two one-dimensional polynomials, i.e. $\theta_{ij}(h, s_-) = \theta_i[2(h - h_{\min})/(h_{\max} - h_{\min}) - 1] \otimes \theta_j[2(s_- - s_{\min})/(s_{\max} - s_{\min}) - 1]$. However, the state space can be conveniently reduced to unidimensional by exploiting the homogeneity properties of the utility and production functions. Indeed, the government's objective is equivalent to $\max_{f/h,\tau,b/h,z} (1+\beta)\log((1-\tau-z)\mathcal{L}(\tau,z) + F(\mathcal{L}(\tau,z)) + (1/R)(h'/h)(b'/h')) + (\phi/n)\log(R(s_-/h) + b/h)$ subject to the public balanced-budget constraints $\tau\mathcal{L}(\tau,z) = n(f/h)$ and $z\mathcal{L}(\tau,z) = (1/n)(b/h)$, the human capital technology $h'/h = A(f/h)^{1-\theta}$ and the private Euler equation $s/h' = (\beta/(1+\beta))(\mathcal{L}(\tau,z)(1-\tau-z) + F(\mathcal{L}(\tau,z))(h/h')) - (1/((1+\beta)R))(b'/h')$. Hence, the policy rules can be approximated using one-dimensional Chebyshev polynomials denoted as $\theta_i(\tilde{s})$ with $\tilde{s} \equiv s_-/h$ confined to the interval $[\tilde{s}_{\min}, \tilde{s}_{\max}] \subseteq \mathbb{R}_+$. The approximated policy rules therefore are $\widetilde{\mathcal{F}}(\tilde{s};\mathbf{a}^f) = \sum_{i=1}^{n} a_i^f\theta_i(\tilde{s})$, $\widetilde{\mathcal{T}}(\tilde{s};\mathbf{a}^\tau) = \sum_{i=1}^{n} a_i^\tau\theta_i(\tilde{s})$, $\widetilde{\mathcal{B}}(\tilde{s};\mathbf{a}^b) = \sum_{i=1}^{n} a_i^b\theta_i(\tilde{s})$, $\widetilde{\mathcal{Z}}(\tilde{s};\mathbf{a}^z) = \sum_{i=1}^{n} a_i^z\theta_i(\tilde{s})$, and $\widetilde{\mathcal{A}}(\tilde{s};\mathbf{a}^s) = \sum_{i=1}^{n} a_i^s\theta_i(\tilde{s})$ where $\mathbf{a}^f = \left(a_1^f, .., a_n^f\right)$, $\mathbf{a}^\tau = (a_1^\tau, .., a_n^\tau)$, $\mathbf{a}^b = \left(a_1^b, .., a_n^b\right)$, $\mathbf{a}^z = (a_1^z, .., a_n^z)$, and $\mathbf{a}^s = (a_1^s, .., a_n^s)$ are vectors of unknown coefficients. We opt for a polynomial of order 15 and show that the errors over 1901 points uniformly distributed over the state space are less than 10^{-10} in all numerical experiments. See Figure 4.

B.4 FINANCIAL SHOCKS

In this appendix, we show the effect of a temporary unexpected financial shock on a PIS. Consider the following experiment. The world interest rate is at its baseline value 1.04^{30} in the initial period. It drops temporarily to 1.03^{30} in the second period and reverts to the baseline

[2] See Judd (1998) for a complete review.

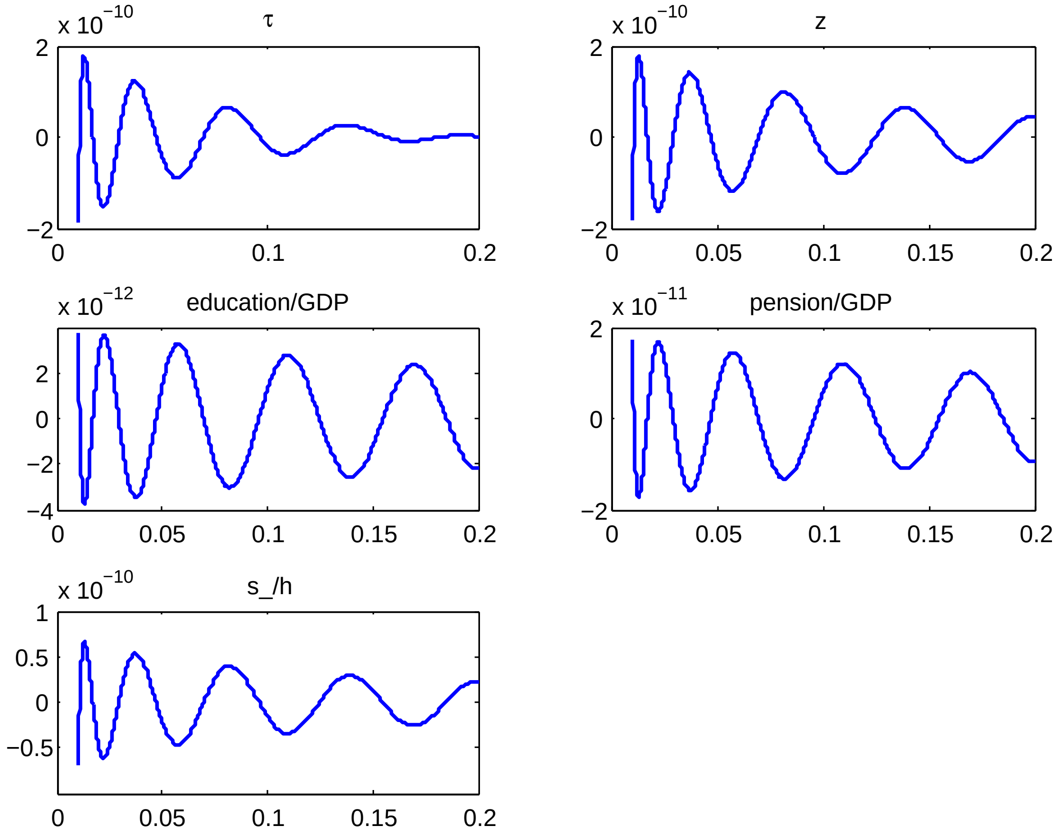

Figure 4: Errors for the government's first-order conditions and the private Euler equation.

value from period $t = 3$ onwards. All the remaining parameter values are as in Table 1. Figure 5 shows the fiscal policy dynamics.

When the unexpected shock hits the economy, it positively affects the present value of future pensions. Indeed, current savings will be capitalized at a lower interest rate, which will lower the opportunity cost of financing retirement benefits. Anticipating this, the government increases both education and pension spending per recipient. More generous government transfers depress private saving and boost the human capital of future generations. The evolution of the asset variables ultimately leads to higher expected pension benefits. In the second period, the world interest rate returns to its initial level and all expectations are realized. Retirement benefits per recipient jump to the rationally anticipated level. An increase of pension contributions generates an adverse effect on the public investment budget and in turn education spending

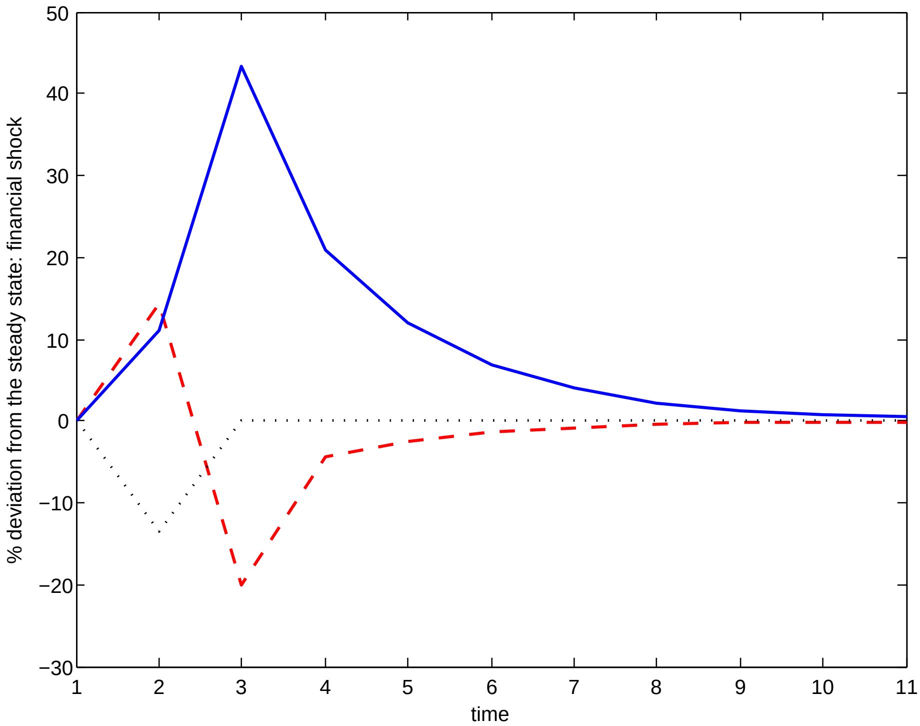

Figure 5: The impulse-response functions of a temporary financial shock to the world interest rate. The solid line represents pensions per retiree and the dashed line depicts public education per student.

per student falls. Moreover, the lower present value of next-period pensions further discourages public investment. From period $t = 3$ onward, pensions per retiree fall and education investment per student grows, converging monotonically to the initial steady state. These predictions are consistent with the empirical evidence of a relationship between the dynamics of the market interest rate and the viability of public fiscal programs (see, e.g., Rodrik, 2011). A more convincing analysis, however, would also take into account the possibility of governments issuing bonds, which may be a worthwhile extension of the paper.

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